Astronomy 596/496 NPA Fall 2009
Problem Set #6
Due: Wednesday, December 2

1. *Geothermal Energy and Geoneutrinos.* Matter in the earth includes various long-lived radioactive species, among then uranium and thorium. Both of these very heavy nuclei decay primarily to lead isotopes, via a series of α and β decays. These decays are geophysically important, as they are an energy source that helps keeps the earth's interior hot so that, e.g., plate tectonics can happen, and volcanos can erupt.

   (a) Natural uranium comes in two isotopes, $^{235}\mathrm{U}$ and $^{238}\mathrm{U}$, but here we need only consider the more abundant U isotope, $^{238}\mathrm{U}$. In this approximation, find the specific energy liberation rate per U atom as follows. Given a mass M_{U} of uranium, find an expression for the rate L_{U} of energy release due $^{238}\mathrm{U} \rightarrow {}^{206}\mathrm{Pb}$ decay, ignoring the small neutrino energy losses. This energy will heat the interior of the earth. Show that the specific energy liberation rate $\epsilon_{\mathrm{U}} = L_{\mathrm{U}}/M_{\mathrm{U}}$ is fixed by the properties of the U decay and is $\epsilon_{\mathrm{U}} \simeq 100\ \mu\mathrm{W\ kg}^{-1}$; similarly show that the decay of $^{232}\mathrm{Th}$ to $^{208}\mathrm{Pb}$ gives $\epsilon_{\mathrm{Th}} \simeq 30\ \mu\mathrm{W\ kg}^{-1}$.

   (b) Around 80–90% of the earth's radioactive heating comes from U-Th decay (with the rest from, e.g., $^{40}\mathrm{K} \rightarrow {}^{40}\mathrm{Ar}$ decay). Assume that all radioactive heating is from U and Th decay, which have average terrestrial mass fractions of $X_{\mathrm{U}} = 0.014\ \mathrm{ppm} = 1.4 \times 10^{-8}$, and $X_{\mathrm{Th}} = 0.052\ \mathrm{ppm} = 5.2 \times 10^{-8}$. Note that the mass of the earth is $M_{\oplus} = 5.97 \times 10^{24}$ kg. Find the total global geothermal heating rate L_{rad} due to U-Th decay. Compare you answer to the earth's net heat outflow rate, measured as $L_{\oplus} \sim 40\ \mathrm{TW} = 40 \times 10^{12}$ W. Comment on the result.

   (c) On average, each $^{238}\mathrm{U}$ decay chain produces $N_{\mathrm{U}} = 6$ neutrinos, and each $^{232}\mathrm{Th}$ decay produces $N_{\mathrm{Th}} = 4.6$ neutrinos. Using the data from (b) and (c), and assuming a spherical earth, find the flux Φ of U and Th decay neutrinos coming from the earth–calculate both the flux from each isotope separately, and then the total. These have become known as "geoneutrinos."

   (d) Imagine that it is possible (say from measuring the neutrino energy spectra) to *separately* measure the neutrinos from $^{238}\mathrm{U}$ and $^{232}\mathrm{Th}$ decay. Show that, under the assumptions of (b) and (c), the geoneutrino flux Φ_i for each isotope $i \in ({}^{238}\mathrm{U}, {}^{232}\mathrm{Th})$ is directly proportional to the terrestrial radioactive heat flux $F_i = L_{i,\mathrm{rad}}/4\pi R_{\oplus}^2$ due to that isotope, and find the constant of proportionality.

   More realistically, we are grateful to even be able to measure the *total* geoneutrino flux Φ_{tot}. Show that the total terrestrial neutrino flux is also proportional to the *total* terrestrial radioactive heat flux F_{tot}, find the constant of proportionality, and comment on how it relates to that of the individual isotopes.

   (e) The relationship you have found in (d) allows us to use a measurement of terrestrial neutrinos to directly test the theory that radioactive decay is an important heat source for the Earth's core, just as measurements of solar neutrinos have tested the theory that fusion is the dominant heat source at the Sun's core. In other words, neutrinos can be used not only for astrophysics but also for geophysics! In fact KamLAND recently reported a measurement of the geoneutrino flux. Arkai et al (KamLAND Collaboration) *Nature* **436**, 499 (2005) find evidence for nonzero geoneutrino flux, but report a 99% CL *upper limit* to the flux of $\Phi_{\rm tot} < \Phi_{\rm max} = 1.62 \times 10^7\ {\rm cm}^{-2}\ {\rm s}^{-1}$. Use this limit and the proportionality you found in (d) to compute the corresponding upper limit geothermal heat flux resulting from U and Th decay. Compare this to the geologically measured value $L_\oplus \sim 40\ {\rm TW} = 40 \times 10^{12}$ W, and comment.

2. *The* s-*Process.*

   (a) Find three (or more) shielded *s*-process nuclei all of which lie between two magic N peaks, and preferably have masses that are close to each other. You will need a chart of the nuclides to do this; one is available online (see links). For each of your nuclei, indicate which nucleus provides the shielding.

   (b) For each of the shielded nuclei from part (a), look up the n-capture cross section (sometimes called the "radiative capture" cross section). You may do this on online (e.g., follow the table of the nuclides or NuDat links). Using the cross sections and the Anders & Grevesse solar abundances found in your handy table, evaluate the accuracy of the local approximation, and comment on your results.

3. *The* r-*Process and Nucleocosmochronology.*

   (a) The simple nature of radioactive decay makes radioisotopes excellent clocks ("nuclear chronometers") if one can measure the decay products or knows something about the initial abundances. For the case of U and Th, the production values are set by the r-process and are fairly well-known. The predicted U/Th production ratio is about $(^{232}{\rm Th}/^{238}{\rm U})_r = 1.8$, and different models give ratios which scatter within about 10% of this value. If the solar r-process elements were all produced in one instantaneous event ("spike"), use the production ratios with the *present* observed abundances from Anders & Grevesse to derive the time elapsed since this spike and the present. Explain why the result of this calculation gives a lower limit to the age of the universe, and comment on the strength of the limit.

   (b) Uranium has been detected in several several halo stars. Most recently Frebel et al *ApJL* **660**, L117 (2007) reported detection of uranium in HE 1523-0901. This is another of the remarkably r-rich halo stars, and is one in which Th has also been detected. Use the observed $({\rm Th/U})_\star = 7.2 \pm 2.0$ ratio to estimate how much older this star is than the earth (i.e., without reference to the theoretical production ratio). Then use the production ratio to derive a limit to the age of the universe, and comment.