Announcements:
• PF 1 posted due this Friday, ≤12noon

Last time: began nuke physics, binding energies

Q: evidence for nuclear physics in solar abs?

Q: why are nuclear interactions complex, compared to E&M?
Nuclear Richness ≡ Complexity
A Look Ahead to Particle Physics

nucleons *not* fundamental particles
but *bound states* of *quarks* and *gluons*
⇒ nuke force really an interaction among complex objects (baryons, mesons) with substructure

Analogy: *Chemistry*
ultimately controlled by E&M,
  but via *atoms*: many-body quantum structures
in principle, can calculate atomic/molecular structure, reactions, scattering *ab initio*
but in practice exceedingly difficult.
Yet can do chemistry anyway:
  notice patterns & useful approximations

take similar approach to nuke physics.
Nuclear Density and Degeneracy

consider nuclear density:
empirically—nearly constant for all nuclei
number density $n = A/Vol = 0.17 \text{ fm}^{-3}$,
or $\rho = M/Vol \simeq 0.2 \text{ GeV fm}^{-3} \simeq 3 \times 10^{14} \text{ g cm}^{-3}$

nucleons are fermions:
must obey Pauli principle $\rightarrow$ important if degeneracy occurs
but does it occur?

$Q$: how to estimate if nuclei are degenerate?
to test for degeneracy ⇒ estimate Fermi energy
i.e., energy that nucleon “gas” would have
if packed together as much as Pauli allows

if degenerate: \( xp_x \sim \hbar \) and also \( y, z \):
if packed into length \( x \), minimal momentum is
“Fermi momentum” \( p_F \sim \hbar / x \)

so for nucleus with size \( r \sim 1A^{1/3} \) fm (1 fm = \( 10^{-13} \) cm)
Fermi momentum \( p_F \sim h/r \sim 2\pi \hbar / r \)
Fermi energy \( E_F = p_F^2 / 2m_u \sim 20 - 40 \ A^{-2/3} \) MeV

Q: what should this be compared with?
Q: what do we conclude?
compare to actual nuclear energy level spacings

www: energy level diagram for $^{12}\text{C}$

we find

\[ E_F > E_{\text{nuke level}} \sim 1 \text{ MeV} \quad (1) \]
\[ E_F > E_{\text{EM}} \sim 1.4Z^2\text{MeV}/r_{\text{fm}} \quad (2) \]

i.e., typical nucleon energies are \textit{below} Fermi level

⇒ to zeroth order, the nucleus is a \textit{degenerate} gas of nucleons confined by the strong force

Note: since \( n \sim A/r^3 = \text{constant} \), nuclear radius scales as

\[ r \sim 1.2A^{1/3} \text{ fm} \]
Mass Formula

goal: understand bulk nature of nuclei
binding energy curve, nature of valley of stability

want to know binding energy of nuclides:

\[ B(A, N, Z) = ? \]  \hspace{1cm} (3)

approach: make a rough model of nucleus, use to find functional form; then use mass data to fill in parameters:

“semi-empirical mass formula”
a.k.a., “semi-unbelievable mass formula”

(pioneered by on Wiezäcker → “wise-acre mass formula”)
identify effects:

\[ BE(A, N, Z) = E_v + E_s + E_c + E_{sym} + E_{pair} + E_{shell} \] (4)

- Volume energy: \( E_v \propto V \propto A \):
  \[ E_v = b_v A; \quad b_v \approx 15.5 \text{ MeV} \]
  \text{not} \quad E_v \propto \# \text{ pairs} = A(A - 1) \sim A^2
  \Rightarrow \text{“saturation” due to short-range nuke force}

- Surface effect: fewer neighbors
  \[ E_s \propto -r^2 \]
  \[ E_s = -b_s A^{2/3}; \quad b_s \approx 16.8 \text{ MeV} \]

- Coulomb repulsion: reduces binding
  \[ E_c \sim -Z(Z - 1)e^2/r \]
  \[ E_C = -b_c Z(Z - 1)A^{-1/3}; \quad b_c \approx 0.72 \text{ MeV} \]
so far: “liquid drop model” – ignored quantum effects

- Symmetry: since degen. fermi gas, cheapest when \( N = Z \):
  \[
  E_{\text{sym}} \propto -|\text{excess}| \sim (Z - N)^2/A
  \]
  \[
  \Rightarrow E_{\text{sym}} = -b_{\text{sym}} (Z - N)^2/A; \quad b_{\text{sym}} \simeq 23 \ \text{MeV}
  \]

- Pairing: Pauli \( \rightarrow \) identical nucleons pair off with opposite spins

\[
E_p = \begin{cases} 
  +\delta & N \text{ even } - P \text{ even} \quad (A \text{ even}) \\
  0 & \text{odd-even} \quad (A \text{ odd}) \\
  -\delta & N \text{ odd } - P \text{ odd} \quad (A \text{ even})
\end{cases}
\]

where \( \delta = b_p A^{-3/4} ; \quad a_p \sim 34 \ \text{MeV} \)

(1) odd-odd nuclei rare
(2) odd \( A \) less bound than even \( A \)

\[Q: \text{consequences for SS abs?}\]

www: SS abundances
Nuclear Physics Encoded in Solar Abundances II: 
The Odd-Even Effect

Recall: plotting abundance vs $A$
“zig-zag” is odd-even $A$ effect
⇒ more confirmation that

$nuclear\ physics\ controls\ solar\ abundances$

Finally:
include quantum periodic effects
$E_{shell} = ?$
need model
Nuclear Shell Model

*in atoms:*
  quantum states $\rightarrow$ electronic shells $\rightarrow$ periodic behavior
  for certain "magic $Z$": closed shell $\rightarrow$ tightly bound electrons
  $\Rightarrow$ unusually stable atoms (e.g., noble gases)

*in nuclei:*
  also quantum states
  expect shell behavior, but not necessarily same numerology

for each nucleon:
(1) approximate force by all other nucleons as a central potential
(2) Schrödinger’s eq. $\rightarrow$ energy levels & occupation numbers
(3) filled levels $\rightarrow$ closed shell
  $\rightarrow$ very tight binding
  occur for special values of $N$ and $Z$
  “magic numbers”
Transp: Pagel, Fig 2.2, Infinite well and 3DHO levels

magic numbers:

\( Z = 2, 8, 20, 40, 82 \)

\( N = 2, 8, 20, 50, 82, 126 \)

www: solar abundances vs \( A \) and vs \( N \)

especially stable if doubly magic:

i.e., both \( N \) and \( Z \) are magic

\( ^4\text{He}, \ ^{16}\text{O}, \ ^{40}\text{Ca}, \ ^{90}\text{Zr}, \ ^{208}\text{Pb} \)

Note: because \( ^4\text{He} \equiv \alpha \) doubly magic \( \rightarrow \) very tightly bound

(1) light nuclei which have \( N = Z = \) even

are tightly bound, “\( \alpha \)” nuclei

e.g., \( ^{12}\text{C}, \ ^{16}\text{O}, \ ^{20}\text{Ne}, \ldots, \ ^{40}\text{Ca} \)

www: SS abs

(2) \( A = 5 \) and \( A = 8 \) unstable: decay to \( \alpha + N \) and \( \alpha + \alpha \)

\( \rightarrow \) “mass gaps” at \( A = 5, 8 \) essential for history of universe
Nuclear Decays

Some nuclei unstable $\rightarrow$ spontaneous decay

Three Decay Modes:

$\alpha$: release $^4\text{He} = \alpha$

- e.g., $^{238}\text{U} \rightarrow ^{234}\text{Th} + \alpha$
  
  decay via nuclear interaction, *nucleons reshuffled*

$\beta$: $(N, Z) \rightarrow (N \pm 1, Z \pm 1) + e + \nu$  \hspace{1cm} (e = $\beta$)

- decay via weak interaction: *nucleon type changed, $\nu$ present*

  - e.g., $\beta^-$: tritium decay $^3\text{H} \rightarrow ^3\text{He}^2 + e^- + \bar{\nu}_e$

  \hspace{1cm} ($\bar{\nu}_e$ = anti-neutrino)

  - i.e., have $n \rightarrow p + e^- + \bar{\nu}_e$

  - $\beta^+$ example: $^{17}\text{F} \rightarrow ^{16}\text{O} + e^+ + \nu_e$

  \hspace{1cm} i.e., have $p \rightarrow n + e^+ + \nu_e$
γ: de-excite, emit photon γ

decay via EM int. \textit{(same nucleus)}

e.g., excited \(^{26}\text{Mg}^*\rightarrow^{26}\text{Mg}^{\text{g.s.}} + \gamma \)

\(E_{\gamma} = E_{\text{excited}} = 1.8 \text{ MeV}\)

\textbf{Q: which decays change } A \text{?}

\textbf{Q: which decays change } Z \text{?}

\textbf{Q: } \beta^- \text{ path on chart of nuclides? } \beta^+ \text{?}