Astro 596/496 NPA

Lecture 4

Aug. 31, 2009

Announcements:

• PF 1 posted due this Friday, $\leq$12noon

Last time: began nuke physics, binding energies

Q: evidence for nuclear physics in solar abs?

Q: why are nuclear interactions complex, compared to E&M?

Nuclear Richness $\equiv$ Complexity

A Look Ahead to Particle Physics

nucleons *not* fundamental particles
  but *bound states* of quarks and gluons
  $\Rightarrow$ nuke force really an interaction among complex objects
  (baryons, mesons) with substructure

Analogy: *Chemistry*
  ultimately controlled by E&M,
    but via *atoms*: many-body quantum structures
  in principle, can calculate atomic/molecular
  structure, reactions, scattering *ab initio*
  but in practice exceedingly difficult.
  Yet can do chemistry anyway:
    notice patterns & useful approximations

take similar approach to nuke physics.

Nuclear Density and Degeneracy

consider nuclear density:
empirically–nearly constant for all nuclei
number density $n = A/\text{Vol} = 0.17\ \text{fm}^{-3}$,
or $\rho = M/\text{Vol} \simeq 0.2\ \text{GeV fm}^{-3} \simeq 3 \times 10^{14}\ \text{g cm}^{-3}$

nucleons are fermions:
must obey Pauli principle $\rightarrow$ important if *degeneracy* occurs
but *does* it occur?

Q: how to estimate if nuclei are degenerate?

to test for degeneracy $\Rightarrow$ estimate Fermi energy
i.e., energy that nucleon "gas" would have
if packed together as much as Pauli allows

if degenerate: $xp_x \sim \hbar$ and also y, z:
if packed into length x, minimal momentum is
"Fermi momentum" $p_F \sim \hbar/x$

so for nuclus with size $r \sim 1A^{1/3}$ fm (1 fm = 10^{-13} cm)
Fermi momentum $p_F \sim h/r \sim 2\pi\hbar/r$
Fermi energy $E_F = p_F^2/2m_u \sim 20 - 40\ A^{-2/3}$ MeV

Q: what should this be compared with?
Q: what do we conclude?

compare to actual nuclear energy level spacings

`www: energy level diagram for` ^{12}C

we find

$$E_F > E_{\text{nuke level}} \sim 1 \text{ MeV} \quad (1)$$

$$E_F > E_{\text{EM}} \sim 1.4 Z^2 \text{MeV}/r_{\text{fm}} \quad (2)$$

i.e., typical nucleon energies are *below* Fermi level

$\Rightarrow$ to zeroth order, the nucleus is a degenerate gas of nucleons confined by the strong force

Note: since $n \sim A/r^3 =$ constant, nuclear radius scales as

$r \simeq 1.2 A^{1/3}$ fm

Mass Formula

goal: understand bulk nature of nuclei
binding energy curve, nature of valley of stability

want to know binding energy of nuclides:

$$B(A, N, Z) = ? \qquad (3)$$

approach: make a rough model of nucleus, use to find functional form; then use mass data to fill in parameters:
"semi-empirical mass formula"
a.k.a., "semi-unbelievable mass formula"
(pioneered by on Wiezäcker → "wise-acre mass formula")

identify effects:

$$BE(A, N, Z) = E_v + E_s + E_c + E_{sym} + E_{pair} + E_{shell} \qquad (4)$$

• volume energy: $E_{\mathrm{V}} \propto V \propto A$:

$E_v = b_v A$; $\quad b_v \simeq 15.5$ MeV

not $E_v \propto \#$ pairs $= A(A-1) \sim A^2$

$\Rightarrow$ "saturation" due to short-range nuke force

• surface effect: fewer neighbors

$E_s \propto -r^2$

$\rightarrow E_s = -b_s \ A^{2/3}$; $\quad b_s \simeq 16.8$ MeV

• Coulomb repulsion: reduces binding

$E_c \sim -Z(Z-1)e^2/r$

$\rightarrow E_C = -b_c \ Z(Z-1)A^{-1/3}$; $\quad b_c \simeq 0.72$ MeV so far: "liquid drop model" – ignored quantum effects

• Symmetry: since degen. fermi gas, cheapest when $N = Z$:
$E_{sym} \propto -|\text{excess}| \sim (Z - N)^2/A$
$\Rightarrow E_{sym} = -b_{sym}\ (Z - N)^2/A; \quad b_{sym} \simeq 23$ MeV

•Pairing: Pauli → identical nucleons pair off with opposite spins

$$E_p = \begin{cases} +\delta & N \text{ even - } P \text{ even} \quad (A \text{ even}) \\ 0 & \text{odd-even} \quad (A \text{ odd}) \\ -\delta & N \text{ odd - } P \text{ odd} \quad (A \text{ even}) \end{cases} \tag{5}$$

where $\delta = b_p A^{-3/4}; \quad a_p \sim 34$ MeV
(1) odd-odd nuclei rare
(2) odd A less bound than even A

∞

Q: consequences for SS abs?

`www: SS abundances`

Nuclear Physics Encoded in Solar Abundances II:
The Odd-Even Effect

Recall: plotting abundance vs A
"zig-zag" is odd-even A effect
$\Rightarrow$ more confirmation that
nuclear physics controls solar abundances

Finally:
include quantum periodic effects
$E_{shell} = ?$
need model

Nuclear Shell Model

in atoms:

quantum states $\longrightarrow$ electronic shells $\longrightarrow$ periodic behavior

for certain "magic Z": closed shell $\longrightarrow$ tightly bound electrons

$\Rightarrow$ unusually stable atoms (e.g., noble gases)

in nuclei:

also quantum states

expect shell behavior, but not necessarily same numerology

for each nucleon:

(1) approximate force by all other nucleons as a central potential

(2) Schrödinger's eq. $\longrightarrow$ energy levels & occupation numbers

(3) filled levels $\longrightarrow$ closed shell

$\longrightarrow$ very tight binding

occur for special values of N and Z

"magic numbers"

Transp: *Pagel, Fig 2.2, Infinite well and 3DHO levels*
magic numbers:
$Z = 2, 8, 20, 40, 82$
$N = 2, 8, 20, 50, 82, 126$
`www: solar abundances vs` A `and vs` N

especially stable if doubly magic:
i.e., both N and Z are magic
^{4}He, ^{16}O, ^{40}Ca, ^{90}Zr, ^{208}Pb

Note: because $^{4}\text{He} \equiv \alpha$ doubly magic $\rightarrow$ very tightly bound
(1) light nuclei which have $N = Z =$ even
are tightly bound, "α" nuclei
e.g., ^{12}C, ^{16}O, ^{20}Ne, ..., ^{40}Ca
`www: SS abs`
(2) $A = 5$ and $A = 8$ **unstable**: decay to $\alpha + N$ and $\alpha + \alpha$
$\rightarrow$ **"mass gaps" at** $A = 5, 8$ essential for history of universe

Nuclear Decays

Some nuclei unstable → spontaneous decay

Three Decay Modes:

α: release ^{4}He $= \alpha$

e.g., ^{238}U→ ^{234}Th $+ \alpha$

decay via nuclear interaction, *nucleons reshuffled*

β: $(N, Z) \rightarrow (N \mp 1, Z \pm 1) + e + \nu \quad (e = \beta)$

decay via weak interaction: *nucleon type changed, ν present*

e.g., β^-: tritium decay $^3_2$H^1→ $^3_1$He$^2 + e^- + \bar{\nu}_e$

($\bar{\nu}_e$ = anti-neutrino)

i.e., have $n \rightarrow p + e^- + \bar{\nu}_e$

β^+ example: ^{17}F→ ^{16}O $+ e^+ + \nu_e$

i.e., have $p \rightarrow n + e^+ + \nu_e$

γ: de-excite, emit photon γ
decay via EM int. (*same nucleus*)
e.g., excited $^{26}\text{Mg}^{*} \rightarrow {}^{26}\text{Mg}^{\text{g.s.}} + \gamma$
$E_\gamma = E_{\text{excited}} = 1.8$ MeV

Q: which decays change A?
Q: which decays change Z?
Q: β^- path on chart of nuclides? β^+?