Astro 596/496 NPA
Lecture 5
Sept. 2, 2009

Announcements:

- PF 1 due Friday ≤12noon
  see tips on next slide
- World-famous cosmologist in the house!
  Paul Steinhardt, Princeton
  "The Endless Universe"
  Physics Colloquium tomorrow, 4pm Loomis 141

Last time: nuclear structure and decays

Q: decay modes?

Q: radioactive decay time behavior?

Tips for PF1 Optional Reading (Asplund et al 2005)

Abundance Notation

very commonly used

$$[A/B] \equiv \log_{10}\left[\frac{(A/B)_{\text{observed}}}{(A/B)_{\text{solar system}}}\right] \tag{1}$$

e.g.,: if a star has $\text{Fe/H}_\star = 0.01\text{Fe/H}_\odot$, then $[\text{Fe/H}]_\star = -2$
note: $[A/B]$ is a *logarithmic* measure of abundance
i.e., $[A/B]$ is a "**d**ecimal **ex**ponent
→ really dimensionless, but "units" sometimes called "`dex`"

also used

$$[A] = 12 + \log_{10}(A/\text{H}) = \log_{10}(10^{12} A/\text{H}) \tag{2}$$

e.g., since $(\text{Fe/H})_\odot = 3 \times 10^{-5}$, then $[\text{Fe}] = 7.5$
also dimensionless logarithmic measure,
"units" sometimes also called "`dex`"

Radioactive Decay Rate

const decay probability P per unit time:

$$\frac{dP}{dt} = \lambda = const \tag{3}$$

and so

$$\frac{dn}{dt} = -n \frac{dP}{dt} = -\lambda n \tag{4}$$

decay lifetime ("*mean life*") $\tau = 1/\lambda$

solution: $\boxed{n = n_0 e^{-t/\tau}}$

also write $n = n_0 2^{-t/t_{1/2}} = n_0 (e^{\ln 2})^{-t/t_{1/2}} = n_0 e^{-t \ln 2 / t_{1/2}}$

"*half-life*" $t_{1/2} = \tau \ln 2$

ω `www: supernova 1987A brightness vs time = ''lightcurve''`

Astrophysical Gamma Decays

famous example: ^{26}Al decay chain

$$\begin{aligned} {}^{26}\mathrm{Al} \rightarrow \; & {}^{26}\mathrm{Mg}^{*} + e + \nu \\ & {}^{26}\mathrm{Mg}^{*} \rightarrow {}^{26}\mathrm{Mg}^{\mathrm{g.s.}} + \gamma \end{aligned} \tag{5}$$

`www:` ^{26}Al `decay scheme`
$t_{1/2}(^{26}\mathrm{Al}) = 0.7$ Myr; $t_{1/2}(^{26}\mathrm{Mg}^{*}) = 0.5$ ps
Q: decay modes? overall timescale?

search sky for 1.8 MeV decay γ rays
`www: Galactic coordinates`
`www: COMPTEL 1.8 MeV sky`

decay line seen!
Q: basic, model-independent implications?
Q: $t_{1/2}(^{26}\mathrm{Al})$ *vs relevant astrophysical timescales–implications?*

^{26}Al Gamma Lines: Smoking Gun of Nucleosynthesis

Most immediately (i.e., most model-independent):

⋆ *live radioactivity is present today in space*
in the form of "fresh" ^{26}Al

But we can push further:

- radioactive decay is exponential in time:
  radionuclides lost after a few $t_{1/2}$
- we know $t_{1/2}(^{26}\text{Al}) = 0.7$ Myr
- age of Earth/Sun ~ 5 Gyr,
  age of Galaxy (i.e., Milky Way disk) ~ 10 Gyr

⇒ the ^{26}Al we see was born "yesterday"

nucleosynthesis is ongoing in the Galaxy today!

Q: implications of ^{26}Al *signal morphology (sky pattern)?*

Implications of ^{26}Al Gamma-Line Sky Morphology

- ^{26}Al signal traces Galactic plane
  this is where star birth (and death) occur

⇒ **(at least some) nucleosynthesis is associated with stars**

- ^{26}Al signal is *diffuse* – not collection of points
  (tricky to show because angular resolution bad)

⇒ ^{26}Al is in the *interstellar medium* (ISM)
and since ^{26}Al short-lived compared to stars

▷ **stars eject fresh nucleosynthesis products which mix into the ISM**

INTEGRAL can measure line shape, position well

`www:` ^{26}Al `line shift vs Galactic longitude`

Q: summarize result? implications? possible interpretation?

Nuclear Reactions

Notation

$A + b \rightarrow c + D = A(b,c)D$

usually $b, c \in p, n, \alpha = {}^4\text{He}, d = {}^2\text{H}, \gamma$

e.g., ${}^{12}\text{C} + {}^4\text{He} \rightarrow {}^{16}\text{O} + \gamma$ is ${}^{12}\text{C}(\alpha,\gamma){}^{16}\text{O}$

Kinematics

preferred frame: center of mass

Diagram

$m_1\vec{v}_1 + m_2\vec{v}_1 = (m_1 + m_2)\vec{V}_{\text{CM}}$ (non-rel)

relative vel: $\vec{v} = \vec{v}_1 - \vec{v}_1$

can show:

$$\begin{aligned} KE &= \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 \\ &= \frac{1}{2}\mu v^2 + \frac{1}{2}(m_1 + m_2)V_{\text{CM}}^2 = (KE)_{\text{in CM}} + (KE)_{\text{of CM}} \end{aligned}$$

where *reduced mass* $\mu = m_1 m_2/(m_1 + m_2)$

Energy conservation

for $a + b \to c + d$

$E_f = E_i$ including rest mass energy!

$(m_c + m_d)c^2 + (KE)_f = (m_a + m_b)c^2 + (KE)_i$

$(KE)_f = (KE)_i + (m_a + m_b - m_c - m_d)c^2 = (KE)_i + Q$

with reaction "Q" value:

$$\begin{aligned} Q &= \text{initial masses - final masses} && (6) \\ &= [m_a + m_b - (m_c + m_d)]\, c^2 && (7) \\ &= \Delta_a + \Delta_b - \Delta_c - \Delta_c && (8) \end{aligned}$$

where last line uses mass defects & baryon conservation

exothermic reactions: $Q > 0$ (mass energy released)

- can occur for any $(KE)_i$

endothermic reactions: $Q < 0$

- need $(KE)_i > |Q|$ to go:
- $\Rightarrow$ there is a "*threshold*" energy

Reaction Physics

classify reactions according to timescale τ:

Direct: $\tau \simeq t_{\text{cross}} = r_{\text{nuke}}/v$

retain "memory" of initial states

"single step" formation of product

Compound: $\tau \gg t_{\text{cross}}$

"forget" initial state

form intermediate QM state (resonance)

"compound" nucleus $A + b \rightarrow W \rightarrow c + D$

decays, emitting particles of final state