Astro 596/496 NPA
Lecture 6
Sept. 4, 2009

Announcements:
- PF 1 was due today
- Problem Set 1 available, due next Friday, Sept. 11

Last time: nuclear decays, nuclear reactions

Q: what determines when a decay or reaction can occur?

Today: nuclear reaction rates
- cross sections defined
- experiments
- reaction rates: basic formalism
- thermonuclear rates

Reaction Rates and Cross Sections

Reaction: $a + b \rightarrow c + d$

Consider particle beam:
"projectiles," number density n_a
incident w/ velocity v
on targets of number density n_b

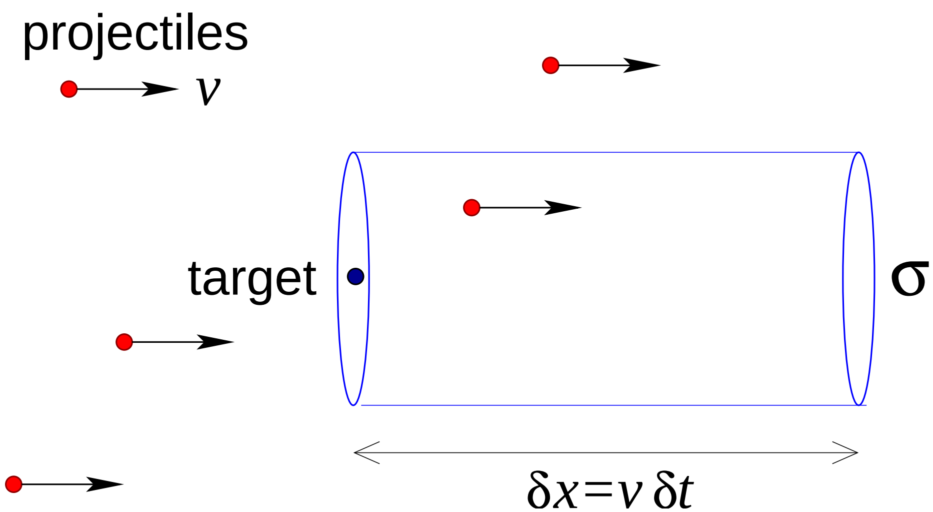
Due to interactions, targets and projectiles "see" each other as spheres of projected area $\sigma(v)$: the

cross section

⋆ fundamental measure interaction strength/probability
⋆ *nuke & particle physics meets astrophysics via* σ

in time δt, what is avg # collisions on one target?
Q: what defines "interaction zone" around target?

interaction zone: particles sweep out "scattering tube"
of area σ, length $\delta x = v\delta t$



interaction volume around target:
$\delta V = \sigma \delta x = \sigma v \delta t$

ω collide if a projectile is in the volume

Cross Section, Flux, and Collision Rate

in δV, # proj $= \mathcal{N}_{\rm proj} = n_a \delta V$
so ave # collisions in δt:

$$\delta \mathcal{N}_{\rm coll} = \mathcal{N}_{\rm prj} = n_{\rm a} \sigma v \delta t \tag{1}$$

so $\delta \mathcal{N}_{\rm coll}/\delta t$ gives

avg collision rate *per target* b $\boxed{\Gamma_{{\rm per}\, b} = n_a \sigma_{ab} v = \sigma_{ab} j_a}$ (2)

where $j_a = n_a v$ is incident flux
Q: Γ units? sensible scalings n_a, σ, v*? why no* n_b*?*

Q: average target collision time interval?
Q: average projectile distance traveled in this time?

estimate avg time between collisions on target b:

mean free time τ

collision rate: $\Gamma = d\mathcal{N}_{\text{coll}}/dt$

so wait time until next collision set by $\delta N_{\text{coll}} = \Gamma_{\text{per}\, b}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{\text{per}\, b}} = \frac{1}{n_a \sigma v} \tag{3}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\text{mpf}} = v\tau = \frac{1}{n_a \sigma} \tag{4}$$

no explicit v dep, but still $\ell(E) \propto 1/\sigma(E)$

Q: physically, why the scalings with n, σ?

Q: what sets σ for billiard balls?

Q: what set σ for $e^- + e^-$ scattering?

Cross Section vs Particle "Size"

if particles interact only by "touching"
(e.g., billiard balls)
then $\sigma \leftrightarrow$ particle radii

but: if interact by force field
(e.g., gravity, EM, nuke, weak)
cross section σ *unrelated* to physical size!

For example: e^- has $r_e = 0$ (as far as we know!)
but electrons scatter via Coulomb (and weak) interaction
"touch-free scattering"

Q: how to measure cross sections experimentally?

Nuclear Astrophysics Experiments

www: NSCL at Michigan State

www: TRIUMF, cyclotron

www: RIKEN

www: LUNA, Gran Sasso

Reaction Rate Per Volume

recall: collision rate *per target* b is $\Gamma_{\text{per}\,b} = n_a \sigma_{ab} v$
total collision rate *per unit volume* is

$$r = \frac{dn_{\text{coll}}}{dt} = \Gamma_{\text{per}\,b} n_b = \frac{1}{1+\delta_{ab}} n_a n_b \sigma v \tag{5}$$

Kronecker δ_{ab}: 0 unless particles a & b identical
Note: *symmetric* w.r.t. the two particles

What if particles have more than one relative velocity?

Reaction Rates: Velocity Distributions

If $v \in$ distribution, rates is average over velocities:

$$\langle r \rangle = \langle n_1 n_2 \sigma v \rangle \tag{6}$$

where $\langle \cdots \rangle$ is avg over relative velocities

given $n_i = \int d^3\vec{v}\ f_i(\vec{v})$,
i.e., $f(v)$ is the velocity distribution of i
and $v_{\mathrm{rel}} = |\vec{v}_2 - \vec{v}_1|$

$$\langle r \rangle = \int d^3\vec{v_1} \int d^3\vec{v_2}\ f(\vec{v}_1)\ f(\vec{v}_2)\sigma(v_{\mathrm{rel}})v_{\mathrm{rel}} \tag{7}$$

Q: (astro)physically relevant velocity distribution(s)?

Thermal Velocity Distribution

Important special case: Non-Relativistic, dilute gas
$\rightarrow$ distribution $f(\vec{v})$ is Maxwell-Boltzmann

at temperature T

$$f_{\mathrm{MB}}(\vec{v})d^3\vec{v} = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right)\left(4\pi v^2 dv\right) \qquad (8)$$

a Gaussian

So: $r = \int \mathrm{gaussian}_1 \times \mathrm{gaussian}_2$
PS 1.3: reduce this to *one* Gaussian
in relative velocity:

$$\langle r \rangle = n_1 n_2 \langle \sigma v \rangle \qquad (9)$$

average over relative v and reduced mass μ

$$\langle\sigma v\rangle = \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int d^3\vec{v}\ v\ \sigma(v) e^{-\mu v^2/2kT}$$
$$= \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty dE\ E\ \sigma(E) e^{-E/kT}$$

"*thermonuclear*" reaction rate

Note $e^{-E/kT}$ factor

suppresses $E \gtrsim kT$ exponentially strongly

Recall Coulomb barrier:

$E_C = Z_1 Z_2 e^2/r = Z_1 Z_2$ 1.44 MeV (1 fm$/r$)

Classically, need $\gtrsim 1$ MeV to overcome barrier

But $kT =$ 0.86 keV $(T/10^7$ K$) \ll E_C$ for solar-like temp !?!

Q: What does this seem to imply?

Q: What's the flaw?