Astro 596/496 NPA

Lecture 7

Sept. 9, 2009

Announcements:

- Problem Set 1 due next time

  Note: intensity units in figure are 10^{-3} photons $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$

  for example: red $\rightarrow \sim 1.2\times10^{-3}$ photons $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$
- TA Office Hours: 3-4pm Thursdays

  ...or by appointment

Last time: nuclear reactions and cross sections

Q: how is a cross section defined*?*

Q: what is the physical significance of σ? units?

Q: how are cross sections related to particle sizes?

Cross Sections: Post-Holiday Reminder

For reaction $a + b \to$ something cross section *defined* as

$$\sigma_{ab} \equiv \frac{\text{reaction rate per target}}{\text{projectile flux}} = \frac{\Gamma_{\text{per}\, b}}{j_a} = \frac{\Gamma_{\text{per}\, a}}{j_b} \qquad (1)$$

where the *flux* of species i with speed v is

$$j_i = n_i v = \# \text{ particles area}^{-1} \text{ time}^{-1} \qquad (2)$$

Physically: cross section σ_{ab} is projected size of
"bullseye" each reactant "sees" in the other
$\Rightarrow$ measures reaction probability/strength

Cross section controlled by physics of $a + b$ interaction
- if short-ranged "contact" interaction: $\sigma \sim$ (particle size)^2
- if long-ranged interaction: σ unrelated to particle size

Q: σ when distribution of speeds? most important example?

Reaction Rates for Thermal Particles

for $ab \to cd$, reaction rate per volume: $r_{ab\to cd} = n_a n_b \langle \sigma_{ab} v \rangle$
where $\langle \cdots \rangle$ is average over distribution of relative v

For non-relativistic gas at temperature T:
relative speeds are in *Maxwell-Boltzmann* distribution

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty dE \ E \ \sigma(E) \ e^{-E/kT} \tag{3}$$

Note $e^{-E/kT}$: exponentially suppresses energies $E \gtrsim kT$

Recall Coulomb barrier:
$E_C = Z_1 Z_2 e^2/r = Z_1 Z_2$ 1.44 MeV (1 fm$/r$)
Classically, need $E \gtrsim 1$ MeV to overcome barrier
But $kT =$ 0.86 keV $(T/10^7$ K$) \ll E_C$ for solar-like temp !?!

Q: What does this seem to imply?
Q: What's the flaw?

Quantum Mechanics to the Rescue

nuclei are quantum particles $\rightarrow$ can tunnel
Probability to tunnel under Coulomb barrier:

$$P \propto e^{-2\pi Z_1 Z_2 e^2/\hbar v} = e^{-bE^{-1/2}} \tag{4}$$

Also: geometrical factor: cross section $\sigma \propto \lambda_{\rm deB}^2$,
$\lambda_{\rm deB} = \hbar/p$ de Broglie wavelength
$\Rightarrow \sigma \propto 1/p^2 \propto 1/E$

expect σ functional form

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta} = \frac{S(E)}{E} e^{-bE^{-1/2}}$$

$S(E)$: "astrophysical S–factor"

- $S(E)$ encodes nuclear contribution to reaction
- $S(E)$ often slowly varying with E
  Q: if so, σ behavior at large E? small E?
- σ and S-factor for ^{3}He$(\alpha,\gamma)^7$Be

Thermonuclear Rates

So: thermonuclear rates reduce to:

$$\begin{aligned} \langle \sigma v \rangle &= \langle \sigma v \rangle_T \\ &= \sqrt{\frac{8}{\pi \mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty dE \; S(E) e^{-E/kT - bE^{-1/2}} \end{aligned}$$

Procedure:

(1) astro theory/obs identifies needed reaction

(2) nuclear expt: measure $\sigma(E) \to S(E)$

(3) find $\langle \sigma v \rangle$ vs T (usually numerically)

(4) fit result to function

Q: note exponential–behavior vs E? implications?

The Gamow Peak

integrand $S(E)e^{-G(E)}$ peaks at/near *minimum* of exponential

$G(E) = E/kT + bE^{-1/2}$

min at $G' = 0$: "most effective energy" or "*Gamow Peak*"

$E_0 = (bkT/2)^{2/3}$, where

$$G_{\rm min} \equiv \tau = G(E_0) = 3(b^2/4kT)^{1/3} \tag{5}$$

$$= 4.25(Z_1^2 Z_2^2 A)^{2/3} \left(\frac{10^9 \text{ K}}{T}\right)^{1/3} \tag{6}$$

Q: behavior with T? interpretation?

expand exponential around peak energy E_0:

$$G(E) \approx \tau + \frac{1}{2} G''(E_0)\,(E - E_0)^2 \tag{7}$$

use expansion of exponential
$G(E) \approx \tau + \frac{1}{2} G''(E_0)\,(E - E_0)^2$
in thermonuclear integral (method of steepest descent)

Then we have

$$\begin{aligned} \langle \sigma v \rangle &\simeq \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} S(E_0) e^{-\tau} \int_{-\infty}^{\infty} dE \; e^{-(E-E_0)^2/2\Delta^2} \\ &= \frac{8}{9\sqrt{3}} \frac{\hbar}{\pi Z_1 Z_2 e^2 m} \tau^2 e^{-\tau} S(E_0) \\ &\propto T^{-2/3} e^{-a/T^{1/3}} \end{aligned}$$

Q: behavior at high T? low T? are these reasonable?

Mean Lifetimes

for reaction $i + j \rightarrow k + l$
define mean lifetime $\tau_i(ij)$ of i against reaction with j as

$$(\dot{n}_i)_{ij} = -\frac{n_i}{\tau_i(ij)} \tag{8}$$

or $\tau_i(ij) = \|n_i/\dot{n}_i\|$

But $\dot{n}_i = -r_{ij} = -n_i n_j \langle\sigma v\rangle_{ij}$
$\Rightarrow$ $\boxed{\tau_i(ij) = 1/n_j\langle\sigma v\rangle_{ij} = 1/\Gamma_{\mathrm{per}\,i}(ij)}$

useful to write

$$\Gamma_{\mathrm{per}\,i}(ij) = n_j\langle\sigma v\rangle_{ij} \simeq \frac{X_j}{A_j}\frac{\rho}{m_u}\langle\sigma v\rangle_{ij} = \frac{X_j}{A_j}\rho[ij] \tag{9}$$

where $[ij] = \langle\sigma v\rangle_{ij}/m_u = N_{\mathrm{Avo}}\langle\sigma v\rangle_{ij}$ given in tabulations

Why would this be a useful form?

Partial Lifetimes: Examples

Reactions in the Sun

In the solar core today:

$T \simeq$ 16 MK $= 1.6 \times 10^7$ K

density $\rho \simeq 150$ g cm^{-3}

$X_{\rm H} \simeq 0.33$

What is lifetime of a deuteron against $d(p,\gamma)^3$He?

$$\begin{aligned} \frac{1}{\tau_d(pd)} &= X_{\rm H}\rho[dp{\to}\gamma^3{\rm He}] \\ &\simeq (0.3)(150\,{\rm g\,cm^{-3}})(2 \times 10^{-10}\,{\rm cm^3\,s^{-1}\,g^{-1}}) \\ &\sim 10^{-8}\ {\rm s^{-1}} \end{aligned}$$

or $\tau_p(pd) \sim 3$ yrs: "immediately" compare $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$

Exponential factor $\tau(E_0) \sim 4\times$ larger!

$\tau_{\mathrm{O}}(p^{16}\mathrm{O}) \sim 10^{57}$ s $\gg$ age of Univ!

no ^{16}O burned in solar core (on main seq)

Particle Physics

Antimatter

Fundamental result of Relativistic QM:
every particle has an antiparticle

- $\overline{e^-} = e^+$ positron
- $\bar{p}$ = antiproton

Fermilab: $p\bar{p}$ collisions

antimatter is **not** second class citizen!
e.g.: e^+ totally *stable* when left alone
So why so volatile in the lab?

www: e^+ annihilation in Galactic center