Astro 596/496 NPA

Lecture 8

Sept. 11, 2009

Announcements:

- Problem Set 1 due
- Preflight 2 posted, due next Friday, noon
- Astronomy Colloquium next Tuesday, here, 4pm:
  William Hanlon, on Ultra-High-Energy Cosmic Rays
  $E_{\text{UHECR}} > 10^{19}$ eV ~ 1 Joule!
  $\rightarrow$ highest-energy particles observed!
  Q: how much more than Fermilab/LHC beams?

Last time: began particle physics overview
  with antimatter

Antiparticle Properties

fundamental symmetries of quantum fields (CPT)
guarantee relationships between
particle χ and antiparticle $\bar{\chi}$ properties:

- mass $m(\bar{\chi}) = m(\chi)$ *not* negative mass!
- decay lifetime $\tau(\bar{\chi}) = \tau(\chi)$
- spin $S(\bar{\chi}) = S(\chi)$
- electric charge $Q(\bar{\chi}) = -Q(\chi)$

sometimes particle = own antiparticle
Q: if so, what must be true?
e.g., $\bar{\gamma} = \gamma$
but: $\bar{n} \neq n$

Conservation Laws

Govern transitions from initial to final states
$\Rightarrow$ scattering, reactions, decays

• (Total) **energy, momentum** conserved
use relativistic definitions, e.g., include rest mass; then

$\sum E_i = \sum E_f$

$\sum \vec{p}_i = \vec{p}_f$

e.g., $n \rightarrow \nu$ $\otimes$ ...since $m_n \neq m_\nu$

• **angular momentum** conserved

$J_i = J_f$

e.g., $n \rightarrow p + e^-$ $\otimes$

J: $s = 1/2 \neq s = 1/2 + s = 1/2 + \ell$

• **electric charge**: $\sum Q_i = \sum Q_f$

e.g., $p + p \rightarrow p + n$ $\otimes$

- **baryon number** conserved

  $B(n) = B(p) = +1$

  $B(\bar{n}) = B(\bar{p}) = -1$

for nucleus, $B_i = A_i \quad \Rightarrow n_{B,i} = A_i n_i$

conservation: $\sum B_i = \sum B_f$

e.g., $p + p \rightarrow p + p + n$ $\otimes$

but $p + p \rightarrow p + p + p + \bar{p}$ OK

- lepton number: lepton $= e$ or ν_e

  $L(e^-) = L(\nu_e) = +1$

  $L(e^+) = L(\bar{\nu}_e) = -1$

conserved: $\sum L_i = \sum L_f$

check: $n \rightarrow p + e^- + \nu_e$ $\otimes$

$n \rightarrow p + e^- + \bar{\nu}_e$ OK

$e^+ e^- \rightarrow \nu_e$ $\otimes$

$e^+ e^- \rightarrow \nu_e \bar{\nu}_e$ OK

Fundamental Particles

high-energy experiments find a zoo of unstable particles
in addition to "everyday" stable species

most of these hundreds of particles
are *strongly* interacting–feel nuclear forces
and are found to be bound states of...

Quarks

n and p *not* fundamental particles
but are composite, have substructure

but: free, isolated, individual quarks have *never* been observed!
Q: so why do we even believe they exist?

⋆ magnetic moments of nucleons

e has $\mu_e = 2e\hbar/m_e c$

but: $\mu_p = 2.79\ e\hbar/m_p c$, $\mu_n = -1.91\ e\hbar/m_n c$

⋆ $e - N$ scattering expts show

nucleons do *not* behave as point particles → substructure

but *do* act like systems fo 3 pointlike particles

"quarks" (Gell-Mann: from J. Joyce's *Finnegan's Wake*)

• hadron "spectroscopy" understandable in quark model

two quark types ("flavors") in nucleons: u "up" d "down"

protons $p = uud$, neutrons $n = udd$

quark electric charge $Q_u = +2/3$, $Q_d = -1/3$

⇒ so: fundamental charge really is $e/3$

spin $S(u) = 1/2 = S(d)$ (fermions)

Hadrons: Systems of Quarks

hadron = made of quarks = strongly interacting

baryon = qqq triple in bound state

e.g., $p = uud$, $n = udd$

also numerous unstable baryons, e.g.,

$\Delta^{++} = uuu$, $\Delta^{-} = ddd$

`www: PDG baryon listings`

meson = $q\bar{q}$ pair in bound state (decays)

e.g., "pion" $\pi^{+} = u\bar{d}$, $\pi^{-} = \overline{\pi^{+}} = \bar{u}d$

$\pi^{0} = 1/\sqrt{2}(u\bar{u} - d\bar{d})$

$m(\pi^{\pm}) = 140$ MeV, $m(\pi^{0}) = 135$ MeV

decay: $\pi^{0} \rightarrow \gamma\gamma$, $\tau(\pi^{0}) = 8.4 \times 10^{-17}$ s

`www: gamma-ray sky` > 100 `MeV:` $pp \rightarrow pp\pi^{0} \rightarrow \gamma\gamma$

`www: PDG meson listings`

can understand hadron masses ("spectrum" of energy states)
and interaction properties
$\Rightarrow$ ground, excited states of quark systems

example: in terms of quark states
baryons $\Delta^+ = uud$, $\Delta^0 = ddu$ ($m_\Delta \sim 1232$ MeV)
are spin $S = 3/2$ excitations of p, n
"excited states" of nucleon

note: mesons & baryons can and do interact:
e.g., $p + p \rightarrow p + n + \pi^+$

Particle Families

Useful to group normal matter constituents as "family"

$$\begin{pmatrix} q_{+2/3} \\ q_{-1/3} \\ \ell_{-1} \\ \ell_0 \end{pmatrix} = \begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \tag{1}$$

High-Energy expts show: other quarks, leptons exist!

★ strange quark s: $Q(s) = -1/3$

$\rightarrow$ strange baryons $\Lambda = uds$, mesons $K^- = s\bar{u}$

★ mu-lepton (muon) μ:

$m(\mu) = 105.7$ MeV $\simeq 200 m_e$

I. Rabi: "Who ordered that?"

`www: PDG lepton listings`

new particles decay to "first family" particles; e.g., $\Lambda \rightarrow p + \pi^-$

Q: implications for early universe? for dark matter?

Periodic Table of Elementary Particles

known fundamental particles: 3 families

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{matrix} \text{charm quark} \\ \\ \\ \\ \end{matrix} \quad \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{matrix} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \\ \end{matrix} \tag{2}$$

+antiparticles

all of these are spin-1/2: **matter is made of fermions!**

note: for quarks and charged leptons,
masses increase with each family
$\rightarrow$ is this same for νs??

Generalized Conservation Laws

Conservation laws: as before, but now

baryon number: includes quarks: $B_q = 1/3$

e.g., $B(\Lambda) = 1$, $B(q\bar{q}) = 0$

$\rightarrow$ “meson number” not conserved

lepton number:

separately conserved for each family (but see discussion of ν oscillations)

e, μ, and τ lepton number each conserved

e.g., $\mu^- \rightarrow e^- + \gamma$: $\otimes$! L_μ, L_e non-cons

instead $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ OK

Whenever see a reaction:

first task is to ensure conservation laws obeyed

Fundamental Interactions: Overview

Interaction	Field Quantum	Mass $m_{\text{boson}}c^2$ (GeV)	Range (cm)	Relative Strength	Typical Cross section at 1 GeV (cm^2)
Strong	Gluon	0	$\sim 10^{-13}$	~ 1	$\sim 10^{-26}$
Weak	$W^{\pm}, Z^0$	82, 91	$\sim 10^{-16}$	$\sim 10^{-5}$	$\sim 10^{-40}$
Electromagnetic	photon	0	∞	$\alpha = 1/137$	$\sim 10^{-29}$
Gravitation	graviton(?)	0	∞	$\sim 10^{-38}$	N/A

Fundamental Interactions and Forces

at quantum level, forces transmitted by boson exchange

e.g., Coulomb scattering: $e\mu \rightarrow e\mu$
exchange photon

boson mass $\leftrightarrow$ interaction range
exchange timescale $m_{\text{boson}} c^2 \tau \lesssim \hbar$
$\Rightarrow$ range $r \lesssim c\tau \sim \hbar / m_{\text{boson}} c$ (*Compton wavelength*) n

- EM: photon $m_\gamma = 0 \rightarrow$ infinite range
  $V_{\text{EM}}(r) \sim 1/r$, so $V \neq 0$ for $r < \infty$
- Gravity: also $V \sim 1/r \rightarrow$ massless graviton(??)
- Weak interaction: massive bosons
  $W^\pm, Z^0$, $M \gg m_p \rightarrow$ finite range

• strong interaction: felt by quarks

key: $V_{\text{strong}}(r) \sim a/r + kr$

as $r \rightarrow \infty$, $V \rightarrow \infty$!

"confinement" : no free quarks found!

always bound into baryons (qqq) mesons ($q\bar{q}$)

Note: nuclear force $\simeq \pi$ exchange

range $r_{\text{nuke}} \sim \hbar/m_{\pi}c \sim 1$ fm

Who feels what?

all particles subject to gravity, and

neutrinos "feel" only weak interaction

charged leptons feel only weak and EM

quarks feel all forces

Note: β decay really quark transformation

$n \rightarrow p + e^- + \bar{\nu}_e$

$udd \rightarrow uud + e^- + \bar{\nu}_e$

$\Rightarrow d \rightarrow u + e^- + \bar{\nu}_e$

Director's Cut Extras

Relativistic Kinematics

Special relativity:
given two events separated by $dX = (dt, dx, dy, dz)$
interval $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ is
invariant: same value for all observers
massless particles (e.g., γ): $ds^2 = 0$

Lorentz transform (boost):
if know $X^\mu = (ct, \vec{x})$ one observer, what is it for another (X')?
$X' = [\gamma(ct - \beta x), \gamma(x - \beta ct), y, z]$
where $\beta = v/c$, $\gamma = 1/\sqrt{1 - v^2/c^2}$

formally similar to spatial rotations
→ coordinates "mix" in linear combo
but Lorentz mixes space and time

Objects which transform this way: 4-vectors

$\Rightarrow$ energy-momentum: $\boxed{P = (E_{\text{tot}}, c\vec{p})}$ is 4-vec

conservation $\rightarrow \sum P_i = \sum P_f$ (same frame)

$\rightarrow \sum E_i = \sum E_f$ and $\sum \vec{p}_i = \sum \vec{p}_f$

include rest mass energy!

note: $P_i^2 = P_f^2$ invariant for any i, f frames

where $P_1 \cdot P_2 = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2$

particle at rest: $P = (mc^2, 0)$

boost with vel $\vec{v}$:

$P' = (\gamma mc^2, \gamma\vec{\beta}mc^2)$

$\Rightarrow$ $E = \gamma mc^2$, and $\vec{p} = \gamma m\vec{\beta}c = \gamma m\vec{v}$

note: invariant $P^2 = E^2 - \vec{p}^2 = m^2 = const$ OK

$\vec{v}/c = c\vec{p}/E$ (when E includes rest mass)

kinetic energy: $T = E - mc^2 = (\gamma - 1)mc^2$

Massless particles:

$m^2 = 0 = E^2 - (cp)^2 \Rightarrow E = cp$

Natural Units

Fundamental dimensionful constants set natural scales
natural to use these as *standards*

- simplifies notation
- very common in particle, nuclear, cosmo literature

fundamental speed limit set by c

- so write all speeds as fraction of this

$\longrightarrow$ effectively set $c = 1$
i.e., all v in terms of c
$E^2 - p^2 = m^2$, $v = p/E$

sometimes also use fundamental angular momentum $\hbar$:
effectively set $\hbar = 1$

helpful conversion: $\hbar c \simeq 200$ MeV fm
e.g., Compton wavelength
$r_c = 1/m = \hbar/mc = \hbar c/mc^2 = 200 \text{ fm}/m_{\text{MeV}}$