Astro 596/496 NPA

Lecture 10

Sept. 16, 2009

Announcements:

- Preflight 2 due Friday noon
- program note: Director's Cut Extras sometimes appear at end of lecture notes

Last time: began cosmology

Q: large-scale structure of the Universe?

Q: cosmological principle?

Cosmic Kinematics

1920's: Hubble, Slipher: all galaxies' spectral lines shifted:

- galaxies move wrt us!
- essentially all galaxies show shift to *red*:

$\lambda_{\rm obs} > \lambda_{\rm lab} = \lambda_{\rm rest}$

Define: **redshift**

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{\rm obs} - \lambda_{\rm emit}}{\lambda_{\rm emit}} \qquad (1)$$

if interpret as Doppler (for non-relativistic $v \ll c$)

$$v \approx cz$$

Edwin Hubble (1929)

`www: Hubble PNAS paper`

`www: original, old-school Hubble diagram`

speed-distance correlation: linear

$$v \propto r \tag{2}$$

Hubble: $v = Kr$

but isotropy implies *Q: what?*

Hubble's Law

Hubble: $v = Kr$

isotropy $\Rightarrow$ same K in all directions

modern: Hubble's Law

$$\boxed{\vec{v} = H\vec{r}} \tag{3}$$

at present: time $\boxed{t_0}$ ("sub-0 = today")

measure

$$H_0 \equiv H(t_0) = 73 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1} \tag{4}$$

Hubble parameter or Hubble "constant" *Q: why scare quotes?*

Q: what are dimensions of H?

Structure + Dynamics: Evolution

observe:

- U. homogeneous, isotropic
- Hubble law $\vec{v} = H\vec{r}$

  Q: restate in simple language? Not a trick question...

  `www: artist's conception`

Q: how reconcile?

at least 2 logical possibilities...

1. "Egoist" interpretation: we are at the center of U.

imagine: **explosion** at $t = 0$, centered on us at $r = 0$
- galaxies all fly away from us
- with distribution of const speeds $v_{\rm gal}$

then $r_{\rm gal} = v_{\rm gal} t_{\rm today}$ fastest $\rightarrow$ farthest!

▷ so $v_{\rm gal} = r_{\rm gal}/t_{\rm today} \equiv H_0 r_{\rm gal} \propto r_{\rm gal}$: recover Hubble's law!

▷ can calculate age of Universe as
$t_{\rm today} = t_{\rm H} = 1/H_0 = 14 \times 10^9$ yr $= 14$ Gyr ago
"Hubble time" useful timescale even to non-egoists!
sets scale of $\sim$ "expansion age" of U

limiting speed: c
fastest explosion debris goes farthest $v_{\rm max} = c = H r_{\rm max}$
at characteristic distance: "**Hubble length**"

$$r_{\rm max} = d_H = \frac{c}{H_0} = c t_H = 4200 \text{ Mpc} \qquad (5)$$

useful lengthscale even to non-egoists!
sets $\sim$ size of *observable* U

Egoist/Explosion Model (Milne) is logically possible! But...

Q: give a philosophical reason why we don't believe this?

Q: give a physical reason why this treatment can't be right?

Q: give an observational reason why we don't believe this?

Critiques of Cosmic Egoism

We are at the center of the universe?

Philosophically:
• not Copernican ("principle of mediocrity")

Physically:
• haven't included gravity!

Observationally:
• Milky Way, Local Group don't look special
not what expect from center of explosion
compare supernova $\longrightarrow$ neutron star, black hole

∞

...yet v pattern makes us look special...

Cosmo Principle Constrains Kinematics

consider arbitrary triangle defined by 3 observers at t_0
Hubble law $\rightarrow$ observers in relative motion
$\rightarrow$ at later time t, larger triangle

the claim:
later Δ always similar to original Δ
Q: what are similar triangles?
Q: why must similarity hold?

diagram: triangles, two sides $r(t_0)$, $r(t)$, $s(t_0)$, $s(t)$
Q: connections among r*'s and* s*'s?*

triangles must be **similar**:

$\Rightarrow$ evolution must keep *sides in same ratios*

so must have

$$\frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)} \tag{6}$$

but this rule holds for any triangle

so ratios can depend only on time t:

$$a(t) = \frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)} \tag{7}$$

The Cosmic Scale Factor

We have shown: Cosmo Principle demands
for two "particles" (possibly Galaxies!)
distance evolves according to

$$\vec{\ell}(t) \;=\; \underbrace{a(t)}_{\substack{\text{scale factor}\\ \textit{time varying}}} \quad \underbrace{\vec{\ell}_0}_{\substack{\text{present distance}\\ \textit{fixed once and for all}}} \tag{8}$$

where we are free to choose $a(t_0) = 1$ today, and
$\ell_0 = \ell(t_0)$ is present value ("comoving coordinate")

$a(t)$ must be universal **scale factor**
can depend only on time
but at any t same value everywhere in space

This is huge!
Q: why? What have we proven? What is character of motion?

Cosmic Expansion

the meaning of Hubble Law: Take 2

2. Einstein interpretation:
will see: General Relativity + Cosmo Principle demand
Universe is expanding
all galaxies receding from all others
bold, strange idea!

Q: consistency check–what must expansion explain?

Expansion: Einstein → Hubble

transparency demo: photocopy universe

for two arbitrary observers (e.g., "galaxies")
scale factor gives distances
$\vec{r}(t) = \vec{r}_0 a(t)$
so velocity is: note: "overdot" is time deriv $\dot{x} \equiv dx/dt$

$$\vec{v}(t) = \dot{\vec{r}} = \vec{r}_0 \dot{a} = \frac{\dot{a}}{a}\, a\vec{r}_0 \equiv H(t)\, \vec{r}(t) \qquad (9)$$

$\Rightarrow$ Hubble law!
now interpret "Hubble parameter"
as **expansion rate** $H(t) \equiv \dot{a}/a$

we have shown:
if A sees Hubble's law, then so do (arbitrary) B and C
thus: if *any* observer measure Hubble's law
then *all* observers will measure Hubble's law!

so: Hubble law implies
$\longrightarrow$ *all* galaxies recede according to same law
$\longrightarrow$ no need for center, space has no special points

Moreover: Hubble law is *only* motion
which preserves homogeneity and isotropy
i.e., *any* other motion breaks cosmo principle
...but Hubble law is exactly what's observed!

Q: implications–present, past, future values for a?

present: at t_0, $a(t) = 1$
expanding, so

past: $a(t) < 1$
future: $a(t) > 1$

e.g., at some time in past $a = 1/2$
"galaxies twice as close"

Q: how do cosmic volumes depend on a?
e.g., *Q: when $a = 1/2$?*

Expansion and Areas, Volumes

consider a cube, galaxies at corners

present side length L_0

diagram: cube, label L_0, expansion arrows

$\rightarrow$ cube is "comoving" w/ expansion

draw arrows

volume $V \propto a^3$

$\rightarrow V = L^3 = L_0^3 a^3 = V_0 a^3$

side area $A = A_0 a^2$

`www: raisin cake analogy`

Director's Cut Extras

More Formally, for the GR Cognescenti

relativistic "interval" between nearby "fundamental observers"
$s = (t, \vec{x}) = (t, r, \theta, \phi)$
and $s' = s + ds = s + (dt, d\vec{x}) = s + (dt, dr, d\theta, d\phi)$:
Roberson-Walker line element:

$$\begin{aligned} ds^2 &= dt^2 - a(t)^2 dx^2 \\ &= dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \end{aligned}$$

$\vec{x}$ is fixed, time-indep comoving coord

1. any FO has fixed comoving coords: $d\vec{x}_{\mathsf{fo}} = 0$
   $\Rightarrow$ FO time elaspes as $ds = dt$:
   $\Rightarrow$ t is cosmic time

1. two nearby FO at same t $(dt = 0)$:
   separated by physical distanc $d\ell = a(t)dx$
   $\Rightarrow$ cosmic distances $\ell(t) = a(t)x$