Astro 596/496 NPA
Lecture 11
Sept. 18, 2009

Announcements:

- Preflight 2 was due at noon
  Q: why don't neutrons in nuclei always decay?
  Q: why doesn't the proton decay?
- Problem Set 2 posted, due next Friday in class

Last time:

- Hubble's Law *Q: namely? characteristic scales?*
  `www: data--Hubble the man vs Hubble the telescope`
- cosmic scale factor $a(t)$
  Q: what is it? physical significance? units? value today?
  Q: connection between a and Hubble's law?

Cosmic Expansion and Cosmic Contents

`www:  balloon analogy`

`www:  raisin cake analogy`

Q: what is tricky, imperfect about each analogy?

Q: baryon number density $n_{\rm B}$ *dependence on* a*?*

Q: nonrelativistic mass ("matter") density $\rho_{\rm m}$ *dependence on* a*?*

Q: implications for early universe?

baryon number is *conserved*

...except at *very* high energies/early times?

so in some volume V:

baryon number $\mathcal{N}_\mathrm{B} = n_\mathrm{B} V = const$ fixed

but $V \propto a^3$, so: $\boxed{n_\mathrm{B} \propto a^{-3}}$

similarly, in nonrelativistic limit:

energy conservation $\rightarrow$ mass conservation $\Rightarrow$ $\boxed{\rho_\mathrm{m} \propto a^{-3}}$

definition: to cosmologist, **matter** $\equiv$ *non-relativistic* matter

today: $\rho_\mathrm{matter}(t_0) \equiv \rho_{\mathrm{m},0}$

at other epochs (while still non-relativistic): $\rho_\mathrm{m} = \rho_{\mathrm{m},0}\ a^{-3}$

in Early U: high densities $\rightarrow$ high reaction rates

$\rightarrow$ processes which are unimportant (slow) now

could have been fast

Q: what about expansion effect on relativistic particles–e.g., photons?

Redshifts

quick-n-dirty: wavelengths are lengths! ..it's right there in the name!
$\longrightarrow$ expansion stretches photon $\lambda \Rightarrow \lambda \propto a$

if emit photon at $t_{\rm em}$, then at later times

$$\lambda(t) = \lambda_{\rm emit} \frac{a(t)}{a(t_{\rm em})} \tag{1}$$

if observe later, $\lambda_{\rm obs} = \lambda_{\rm em} \; a_{\rm obs}/a_{\rm em}$
measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1 - a_{\rm em}}{a_{\rm em}} \;\; \Rightarrow \;\; a_{\rm em} = a(z) = \frac{1}{1+z}$$

Scale factor $\leftrightarrow$ redshift

$$a = \frac{1}{1+z}$$
$$z = \frac{1}{a} - 1$$

Example: most distant quasar has $z = 6.4$

`www: SDSS QSO recordholder`

For this quasar:

$\rightarrow$ scale factor $a = 1/(1+6.4) = 0.135$

interparticle (intergalactic) distances 13.5% of today!

$\rightarrow$ galaxies 1+6.4=7.4 times closer

squeezed into volumes $(7.4)^3 = 400$ times smaller!

Q: expansion effect on photon energies?

Redshifts and Photon Energies

in photon picture of light: $E_\gamma = hc/\lambda$

so in cosmological context photons have

$$E_\gamma \propto \frac{1}{a} \tag{2}$$

$\rightarrow \gamma$ energy redshifts

Consequences:

▷ *Q: photon energy density $\varepsilon(a)$?*

▷ if thermal radiation,

Q: $T \leftrightarrow \lambda$ connection?

Q: expansion effect on T?

Relativistic Species

Photon energy density: $\varepsilon = E_\gamma \, n_\gamma$
avg photon energy: $E_\gamma \propto a^{-1}$
photon number density: conserved $n_\gamma \propto a^{-3}$ (if no emission/absorption)
$\Rightarrow$ for relativistic species $\equiv$ **radiation** $\boxed{\varepsilon_{\rm rad} \propto a^{-4}}$

Thermal (blackbody) radiation:
Wien's law: $T \propto 1/\lambda_{\rm max}$
but since $\lambda \propto a \rightarrow$ then $\boxed{T \propto 1/a}$

Consequences:

- $\varepsilon_{\rm rad} \propto T^4$: Boltzmann/Planck!
- T decreases $\rightarrow$ U cools!
  today: CMB $T_0 = 2.725 \pm 0.001$ K
  distant but "garden variety" quasar: $z = 3$
  "feels" $T = 8$ K (effect observed!)

Cosmodynamics

$a(t)$ gives expansion history of the Universe
which in turn tells how densities, temperatures change
$\longrightarrow$ given $a(t)$ can recover all of cosmic history!

but...

How do we know $a(t)$?
Q: What controls how scale factor $a(t)$ *grow with time?*

Cosmodynamics Computed

cosmic dynamics is evolution of a system which is

- gravitating
- homogeneous
- isotropic

Complete, correct treatment: General Relativity

$\rightarrow$ take GR! ...or Cosmology next semester

quick 'n dirty:

Non-relativistic (Newtonian) cosmology

pro: gives intuition, and right answer

con: involves some ad hoc assumptions only justified by GR

Inputs:

- arbitrary cosmic time t
- cosmic mass density $\rho(t)$, spatially uniform
- cosmic pressure $P(t)$: in general, comes with matter but for non-relativistic matter, P not important source of energy and thus mass ($E = mc^2$) and thus gravity so ignore: take $P = 0$ for now (really: $P \ll \rho c^2$)

Construction:
pick arbitrary point $\vec{r}_{\text{center}} = 0$,
center of "comoving" sphere of some radius $r(t)$
which always encloses some arbitrary but fixed mass

$$M(r) = \frac{4\pi}{3}\, r^3 \rho = const \tag{3}$$

a point on the sphere feels acceleration *Q: what?*

Newtonian Cosmodynamics

a point on the sphere feels acceleration

$$\ddot{\vec{r}} = \vec{g} = -\frac{GM}{r^2}\hat{r} \tag{4}$$

with pressure $P = 0$

multiply by $\dot{\vec{r}}$ and integrate:

$$\begin{aligned} \dot{\vec{r}} \cdot \frac{d}{dt}\dot{\vec{r}} &= -GM\frac{\hat{r} \cdot d\vec{r}/dt}{r^2} \qquad (5) \\ \frac{1}{2}\dot{r}^2 &= \frac{GM}{r} + K = \frac{4\pi}{3}G\rho r^2 + K \qquad (6) \end{aligned}$$

Q: physical significance of K? of it's sign?

Friedmann (Energy) Equation

introduce scale factor: $\vec{r}(t) = a(t)\vec{r}_0$

"energy" eqn: Friedmann eq.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\kappa c^2}{R^2 a^2} \tag{7}$$

we will see: full GR gives $K = r_0^2(\kappa c^2/R^2)$

with parameters

- $\kappa = \pm 1, 0$, and
- const R is lengthscale: "curvature" of U.

In full GR:

▷ Friedmann eq. holds even for relativistic matter, but

▷ where $\rho = \sum_{\mathrm{species},i} \varepsilon_i/c^2$: mass-energy density

Q: $a(t)$ behavior if $K = \kappa = 0$? if $\kappa \neq 0$?