Announcements:
- Preflight 2 was due at noon
  \(Q:\) why don’t neutrons in nuclei always decay?
  \(Q:\) why doesn’t the proton decay?
- Problem Set 2 posted, due next Friday in class

Last time:
- Hubble’s Law \(Q:\) namely? characteristic scales?
  \(www:\) data—Hubble the man vs Hubble the telescope
- cosmic scale factor \(a(t)\)
  \(Q:\) what is it? physical significance? units? value today?
  \(Q:\) connection between \(a\) and Hubble’s law?
Cosmic Expansion and Cosmic Contents

www: balloon analogy
www: raisin cake analogy
Q: what is tricky, imperfect about each analogy?

Q: baryon number density $n_B$ dependence on $a$?
Q: nonrelativistic mass ("matter") density $\rho_m$ dependence on $a$?
Q: implications for early universe?
baryon number is *conserved* …except at *very* high energies/early times?

so in some volume $V$:

baryon number $N_B = n_B V = \text{const}$ fixed

but $V \propto a^3$, so: $n_B \propto a^{-3}$

similarly, in nonrelativistic limit:

energy conservation $\Rightarrow$ mass conservation $\Rightarrow \rho_m \propto a^{-3}$

definition: to cosmologist, *matter* $\equiv$ *non-relativistic* matter

today: $\rho_{\text{matter}}(t_0) \equiv \rho_{m,0}$

at other epochs (while still non-relativistic): $\rho_m = \rho_{m,0} \; a^{-3}$

in Early U: high densities $\Rightarrow$ high reaction rates
$\Rightarrow$ processes which are unimportant (slow) now
$\quad$ could have been fast

Q: what about expansion effect on relativistic particles—e.g., photons?
Redshifts

quick-n-dirty: wavelengths are lengths! ..it’s right there in the name!
→ expansion stretches photon $\lambda \Rightarrow \lambda \propto a$

if emit photon at $t_{\text{em}}$, then at later times

$$\lambda(t) = \lambda_{\text{emit}} \frac{a(t)}{a(t_{\text{em}})} \tag{1}$$

if observe later, $\lambda_{\text{obs}} = \lambda_{\text{em}} \frac{a_{\text{obs}}}{a_{\text{em}}}$
measure redshift today:

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} = \frac{1 - a_{\text{em}}}{a_{\text{em}}} \Rightarrow a_{\text{em}} = a(z) = \frac{1}{1 + z}$$
Scale factor ↔ redshift

\[
a = \frac{1}{1 + z} \\
z = \frac{1}{a} - 1
\]

Example: most distant quasar has \( z = 6.4 \)
www: SDSS QSO recordholder

For this quasar:
→ scale factor \( a = 1/(1 + 6.4) = 0.135 \)
interparticle (intergalactic) distances 13.5% of today!
→ galaxies \( 1+6.4=7.4 \) times closer
squeezed into volumes \((7.4)^3 = 400\) times smaller!

Q: expansion effect on photon energies?
Redshifts and Photon Energies

in photon picture of light: \( E_\gamma = \frac{hc}{\lambda} \)

so in cosmological context photons have

\[ E_\gamma \propto \frac{1}{a} \]  \hspace{1cm} (2)

→ \( \gamma \) energy redshifts

Consequences:

▷ Q: photon energy density \( \varepsilon(a) \)?
▷ if thermal radiation,
  Q: \( T \leftrightarrow \lambda \) connection?
  ◦ Q: expansion effect on \( T \)?
Relativistic Species

Photon energy density: $\varepsilon = E_\gamma n_\gamma$

avg photon energy: $E_\gamma \propto a^{-1}$

photon number density: conserved $n_\gamma \propto a^{-3}$ (if no emission/absorption)

$\Rightarrow$ for relativistic species $\equiv$ radiation $\varepsilon_{\text{rad}} \propto a^{-4}$

Thermal (blackbody) radiation:

Wien’s law: $T \propto 1/\lambda_{\text{max}}$

but since $\lambda \propto a \rightarrow$ then $T \propto 1/a$

Consequences:

- $\varepsilon_{\text{rad}} \propto T^4$: Boltzmann/Planck!
- $T$ decreases $\rightarrow$ U cools!
  
  today: CMB $T_0 = 2.725 \pm 0.001$ K
  distant but “garden variety” quasar: $z = 3$
  “feels” $T = 8$ K (effect observed!)
Cosmodynamics

$a(t)$ gives expansion history of the Universe which in turn tells how densities, temperatures change. → given $a(t)$ can recover all of cosmic history!

but...

How do we know $a(t)$?

Q: What controls how scale factor $a(t)$ grow with time?
Cosmodynamics Computed

cosmic dynamics is evolution of a system which is
- gravitating
- homogeneous
- isotropic

Complete, correct treatment: General Relativity
   → take GR! ...or Cosmology next semester

quick ‘n dirty:
Non-relativistic (Newtonian) cosmology
   pro: gives intuition, and right answer
   con: involves some ad hoc assumptions only justified by GR
Inputs:
• arbitrary cosmic time $t$
• cosmic mass density $\rho(t)$, spatially uniform
• cosmic pressure $P(t)$: in general, comes with matter
  but for non-relativistic matter, $P$ not important source of
  energy and thus mass ($E = mc^2$) and thus gravity
  so ignore: take $P = 0$ for now (really: $P \ll \rho c^2$)

Construction:
pick arbitrary point $\vec{r}_{\text{center}} = 0$,
center of “comoving” sphere of some radius $r(t)$
which always encloses some arbitrary but fixed mass

$$M(r) = \frac{4\pi}{3} r^3 \rho = \text{const}$$  \hspace{1cm} (3)

a point on the sphere feels acceleration $Q$: what?
Newtonian Cosmodynamics

a point on the sphere feels acceleration

\[ \ddot{r} = \ddot{g} = -\frac{GM}{r^2} \hat{r} \]  \hspace{1cm} (4)

with pressure \( P = 0 \)

multiply by \( \dot{r} \) and integrate:

\[ \dot{r} \cdot \frac{d}{dt} \dot{r} = -GM\frac{\dot{r} \cdot d\vec{r}/dt}{r^2} \]  \hspace{1cm} (5)

\[ \frac{1}{2} \dot{r}^2 = \frac{GM}{r} + K = \frac{4\pi}{3}G\rho r^2 + K \]  \hspace{1cm} (6)

Q: physical significance of \( K \)? of it’s sign?
Friedmann (Energy) Equation

introduce scale factor: $\vec{r}'(t) = a(t)\vec{r}_0$

"energy" eqn: Friedmann eq.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G \rho - \frac{\kappa c^2}{R^2 a^2}$$

we will see: full GR gives $K = r_0^2(\kappa c^2/R^2)$

with parameters

- $\kappa = \pm 1, 0$, and
- const $R$ is lengthscale: “curvature” of $U$.

In full GR:

▷ Friedmann eq. holds even for relativistic matter, but

▷ where $\rho = \sum_{\text{species},i} \varepsilon_i/c^2$: mass-energy density

Q: $a(t)$ behavior if $K = \kappa = 0$? if $\kappa \neq 0$?