Astro 596/496 NPA
Lecture 16
Sept. 30, 2009

Announcements:
- Preflight 3 due noon Friday

Last time: began big bang nucleosynthesis

Q: what are we trying to find out?
Q: how will the results be expressed?
Q: what are initial conditions, cosmic context?
Q: what are simplest realistic assumptions needed?

Standard Big Bang Nucleosynthesis

Simplest Realistic BBN Theory: **"Standard" BBN**

★ gravity = General Relativity → FLRW universe

★ microphysics = Standard Model of Particle Physics
particle content, couplings as measured in lab
ordinary EM, weak, nuclear interactions

★ neutrinos: $N_\nu = 3$ species with usual left-handed couplings
and no net neutrino number (chem. potential $\mu_\nu / T \ll 1$)

Cosmic Context and Initial Conditions

radiation dominated: $\gamma, \nu\bar{\nu}$ relativistic, also $e^{\pm}$ for $T \gtrsim m_e$
baryons: initially free nucleons n, p

Initially ($T \gtrsim 1$ MeV): weak reactions fast:
drive $n \leftrightarrow p$ interconversion

$$n + \nu_e \leftrightarrow p + e^- \tag{1}$$
$$p + \bar{\nu}_e \leftrightarrow n + e^+ \tag{2}$$

"Fast": rates per particle $\Gamma = n\sigma v \gg H$
or, mean life against rxn $\tau = \Gamma^{-1} \ll H^{-1} \sim t$

Note: since weak interactions fast, EM rxns also fast:
$\Rightarrow$ all particles thermal, w/ same T

while weak interaction is fast, i.e., *in equilibrium*
n/p ratio is “thermal”

think of as *2-state* system: the “nucleon,”
• nucleon “ground state” is the *proton*: $E_1 = m_p c^2$
• nucleon “excited state” is the *neutron*: $E_2 = m_n c^2$
when in equilibrium, Boltzmann sez:

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2 - E_1)/T} = e^{-(m_n - m_n)/T} \tag{3}$$

with $\Delta m = m_n - m_p = 1.293318 \pm 0.000009$ MeV

at $T \gg \Delta m$: $n/p \simeq 1$
at $T \ll \Delta m$: $n/p \simeq 0$

Weak $n \leftrightarrow p$ Rates

example: want rate Γ_n per n of $\nu + n \rightarrow e^- + p$
as func. of T

Generally,

$$\Gamma_n = n_\nu \langle \sigma v \rangle \sim T^3 \langle \sigma \rangle \qquad (4)$$

since $v_\nu \simeq c$ and $n_\nu \sim T^3$

can show: cross section $\sigma \sim \sigma_0 (E_e/m_e)^2 \propto E^2$
where $\sigma_0 \sim 10^{-44}\ \mathrm{cm}^2$ very small!
so thermal avg: $\langle \sigma \rangle \sim \sigma_0 (T/m_e)^2$

for experts: $\sigma \sim G_F^2 T^2 \sim \alpha_{\mathrm{weak}} T^2 / M_W^4$
so $\Gamma_{\mathrm{weak}} \sim \alpha_{\mathrm{weak}} T^5 / M_W^4$

Weak Freezeout

when *in equilibrium*, U completely described by
T + conserved quantum #s (chem potentials μ)

But: U would be *boring* if always in equilibrium

Happily, U out of eq. sometimes $\rightarrow$ **"freeze-out"**
$\Rightarrow$ freeze-outs are most interesting times in cosmology
BBN, CMB, DM, baryon excess: all stem from freezeouts

for BBN: $n \leftrightarrow p$ equilibrium only holds
when weak reactions can maintain it

Q: What would cause equilibrium to fail?
Q: How would you quantify when eq fails?

Cosmic Freezeouts

Rule of thumb: a reaction is

(1) in equilibrium when
conversion rate per nucleon $\Gamma \gg H$ Hub. rate
i.e., mean lifetime $\ll$ expansion time
or, mean free path $\ll$ horizon size $\sim ct \sim cH^{-1}$

(2) "frozen out" when $\Gamma \ll H$

Suggests rough criterion from "freezeout"
• when $\boxed{\Gamma = H}$
i.e., T_{f} set by: $H(T_f) = \Gamma(T_f)$

we'll show this in more detail later...

Weak Freezeout Temperature

Weak interactions freeze when $H = \Gamma_{\text{weak}}$, i.e.,

$$\sqrt{G_{\text{N}}}T^2 \sim \sigma_0 m_e^{-2} T^5 \quad (5)$$

$$\Rightarrow T_{\text{weak freeze}} \sim \frac{(G_{\text{N}})^{1/6}}{(\sigma_0/m_e^2)^{1/3}} \sim \text{1 MeV} \quad (6)$$

gravity & weak interactions conspire to give $T_{\text{f}} \sim m_e \sim B_{\text{nuke}}$!

for experts: note that $G_{\text{N}} = 1/M_{\text{Planck}}^2$, so

$$\frac{T^2}{M_{\text{Pl}}} \sim \alpha_{\text{weak}} \frac{T^5}{M_W^2} \quad (7)$$

$$\Rightarrow T_{\text{freeze}} \sim \left(\frac{M_W}{M_{\text{Pl}}}\right)^{1/3} M_W \sim \text{1 MeV} \quad (8)$$

8 freeze at nuclear scale, but by accident!

Q: what happens to n, p then? what else is going on?

Interlude: Pair Annihilation

right after weak freezeout, T_γ drops below $m_e = 0.511$ MeV
pairs become nonrelativistic, annihilate: $e^+e^- \rightarrow \gamma\gamma$
• mass energy $\rightarrow$ back to radiation
• small leftover amount of e^-

★ a sort of "heating" but really just restores relativistic energy
T_γ never rises, but cooling is briefly slowed

★ since νs decoupled, don't receive pair energy
cooler than photons thereafter
can show: $T_\nu = (4/11)^{1/3} T_\gamma = 0.714 T_\gamma$
today, the (relativistic) cosmic neutrino backgrounds have
$T_{\nu,0} = 0.714 T_{\gamma,0} = 1.95$ K

★ if you can think of how to detect this *cosmic ν background*
let me know and we'll publish–you can even be second author!

The Short but Interesting Life of a Neutron

(1) at $T > T_f$, $t \sim 1$ s
$n \leftrightarrow p$ rapid
maintain $n/p = e^{-\Delta m/T}$

(2) at $T = T_f$,
fix $n/p = e^{-\Delta m/T_f} \simeq 1/6$
so n "mass fraction" is

$$X_n = \frac{\rho_n}{\rho_B} = \frac{m_n n}{m_n n + m_p p} \approx \frac{n}{n+p} \approx 1/7 \tag{9}$$

(3) until nuclei form,
free n decay: $\dot{n} = -n/\tau_n$, with $\tau_n = 885.7 \pm 0.8$ s
then mass fraction drops as

$$X_n = X_{n,i} e^{-\Delta t/\tau} \tag{10}$$

Q: why take this simple from?

Deuterium Bottleneck

Build complex nuclei from n, p

first step: *deuterium production* $n + p \rightarrow d + \gamma$

`www: BBN reaction network`

energy release $Q = B(d) = E_\gamma = 2.22$ MeV: exothermic

reverse "photodissociation" $d + \gamma \rightarrow n + p$ allowed but *endo*thermic

Naïvely: at $T < T_f < Q$, too cold to photo-dis

But: $n_\gamma / n_B = 1/\eta \sim 10^9 \gg 1$

$\Rightarrow$ many photons per baryon

$\Rightarrow \langle E_\gamma \rangle < Q$, but many photons have $E_\gamma > Q$

D can't survive until $T \ll Q$!

c.f. delay in recombination–same idea

Q: How low to go?

Nuclear Statistical Equilibrium

For a NR species (Maxwell-Boltzmann):

$$n = \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-(m-\mu)/T} \tag{11}$$

For $n(p,\gamma)d$ in chem eq: $\mu_n + \mu_p = \mu_d$,
(since $\mu_\gamma = 0$), so

$$\begin{aligned} \frac{n_n n_p}{n_d} &= \left(\frac{(m_n m_p/m_d)kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_n+m_p-m_d)/T} \\ &= \left(\frac{m_u kT/2}{2\pi\hbar^2}\right)^{3/2} e^{-B_D/T} \end{aligned} \tag{12}$$

example of "nuclear statistical equilibrium" this example: Saha equation

Use mole fraction $Y_i = n_i/n_B$ and $n_B = \eta n_\gamma$

$$Y_d \sim Y_n Y_p \eta (T/m_u)^{3/2} e^{B_D/T} \tag{13}$$

Q: what is low-T behavior?

When $Y_d \rightarrow 1$: Nuke buildup starts

$$\ln Y_d \simeq B_D/T + \ln \eta + 3/2 \ln T/m_u \sim 0 \tag{14}$$

so

$$T_D \simeq \frac{B_D}{\ln \eta^{-1}} \sim 0.07 \text{ MeV} \tag{15}$$

i.e., nuke rxns begin at $T \simeq 10^9$ K Note: $T_D \ll B_2$ since $\eta \ll 1$

time $t_d \sim 200$ s $\rightarrow$ "the first 3 min"

between freezeout and T_D:
free n decay: $X_n = X_{n,i} e^{-\Delta t/\tau} \simeq 0.12$

`www: nuke network` *Q: where is flow direction? why?*

Nuke reaction flow $\rightarrow$ highest binding energy $\rightarrow$ ^{4}He

almost all $n \rightarrow ^4$He: $n(^4\text{He})_{\text{after}} = 1/2 \; n(n)_{\text{before}}$

$$Y_p = X(^4\text{He}) \simeq 2(X_n)_{\text{before}} \simeq 0.24 \quad (16)$$

$\Rightarrow \sim 1/4$ of baryons into ^{4}He, 3/4 $p \rightarrow$H
result weakly (log) dependent on η
Robust prediction: **large universal ^{4}He abundance**

But nuke rxns also freeze out
$\Rightarrow n \rightarrow ^4$He conversion incomplete
leftover traces of incomplete burning:

- D
- ^{3}He (and ^{3}H$\rightarrow^3$He)
- ^{7}Li (and ^{7}Be$\rightarrow^7$Li)

trace abundances $\leftrightarrow$ nuke freeze T
$\Rightarrow$ strong $n_B \propto \eta$ dependence

BBN theory: main result

- light element abundance predictions
- depend on baryon density $\leftrightarrow \eta \leftrightarrow \Omega_{\text{baryon}}$

"Schramm Plot' '

Lite Elt Abundances vs η

summarizes BBN theory predictions

`www: Schramm plot`

Note: no $A > 7$... *Q: why not?*

Why don't we go all the way to ^{56}Fe*?*

after all: most tightly bound

$\Rightarrow$ most favored by nuke stat equil