Announcements:
• Preflight 5 posted, due noon Friday

Last time: Solar Neutrino Problems and Solution
Q: what are the problems?
Q: what are the two main classes of solution? (pre-SNO)
Q: how does SNO show the nature of the solution?
Q: what does SNO imply for neutrino physics?
Solar Neutrino Problem(s) Pre-SNO

*observed* \( \nu \) fluxes *less than* Standard Solar Model predictions

- Radiochemical: Chlorine, Gallium
- Water Čerenkov: Super-Kamiokande
  - but \( \nu_{\text{super-}} \) point back to Sun, have expected energy spectrum

**Possible Solutions**

- Standard Solar Model wrong—\( \nu \) flux overpredicted (but \( pp \)?)
- Standard Model of particle physics wrong

**Experimentum Crucis: SNO**

- independently measure \( ^8 \text{B} \ \nu_e \) flux, all-flavor flux
- \( \Phi_{\nu_e}/\Phi_{\text{tot}} = 0.31 \)

\( \Rightarrow \) **large non-\( \nu_e \) flux arriving in detectors!**
Implications: New Neutrino Physics!

The Sun makes only $\nu_e$
Q: why? e.g., why not $\nu_\mu$?
$\rightarrow$ if no new $\nu$ physics, only $\nu_e$ at Earth
$\rightarrow$ predict $\Phi_{CC}(\nu_e) = \Phi_{NC}(\nu_x)$

SNO measures $\Phi_{NC}(\nu_x) > \Phi_{CC}(\nu_e)$!
with very high confidence!

non-$\nu_e$ flux arriving in detector!

A big deal:
• demands new neutrino physics
• indep. of detailed solar model
Triumph of the Standard Solar Model

SNO bonus: can infer total $^8$B $\nu$ flux

compare Bahcall SSM (Bahcall & Pinsonneault 2004):

$$\Phi_{SSM}(^8B) = 5.79(1 \pm 0.23) \times 10^6 \, \nu \, \text{cm}^{-2} \, \text{s}^{-1}$$

$$= [0.88 \pm 0.04(\text{exp}) \pm 0.23(\text{thy})] \Phi_{SNO}^{NC}$$

consistent! SSM working extremely well!

$\Rightarrow$ major triumph for stellar evolution!

woo hoo!

2002 Nobel Prize in Physics: Ray Davis
Interlude: Updike Poem
Solar Neutrino Schizophrenia

total $\nu_e + \nu_\mu + \nu_\tau$ flux *in detectors* agrees with SSM flux *out of solar core*

but solar $\nu$s must *start as $\nu_e$*

→ neutrinos must **transmute** on the way!

i.e., $\nu_e \rightarrow \nu_\mu, \tau$!

there’s more:

<table>
<thead>
<tr>
<th>$\nu_e$ Experiment</th>
<th>$E_{\nu,\text{min}}$ Threshold</th>
<th>Obs/SSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallium</td>
<td>$&gt; 0.233$ MeV</td>
<td>$0.59 \pm 0.06 \pm 0.04$</td>
</tr>
<tr>
<td>Chlorine</td>
<td>$&gt; 0.814$ MeV</td>
<td>$0.33 \pm 0.03 \pm 0.05$</td>
</tr>
<tr>
<td>Super-K</td>
<td>$&gt; 5$ MeV</td>
<td>$\sim 0.4$</td>
</tr>
</tbody>
</table>

⇒ transmutations must be energy-dependent:

Q: what should dependence be like?

www: solar nu spectrum
Solar Neutrino Transformation Properties

Need:
• small $\nu_e$ suppression at low energies ($pp$: $\lesssim 0.4$ MeV)
• large $\nu_e$ suppression ($>50\%$) at higher energies

Non-trivial neutrino physics required!
If neutrinos have nonzero mass
• family status \((e, \mu, \tau \text{ “flavor”})\), and
• mass
can be distinct!

\(\nu\) family \(\rightarrow\) lepton number conservation in Weak interactions
formally, \(\nu\)s couple to Weak interaction as
**flavor eigenstates**
flavor basis vectors \(|\nu_\alpha\rangle\), \(\alpha = e, \mu, \tau\)

free (vacuum) neutrino \(\rightarrow\) propagates as
**mass eigenstate**
mass basis vectors \(|j\rangle\), \(j = 1, 2, 3\)
Basis Transformation: Flavor/Weak ↔ Mass/Vacuum

Key idea: \textbf{mass eigenstate} \neq \textbf{flavor eigenstate}

analogous to spin-$\frac{1}{2}$: $S_z$ eigenstates $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ vs $S_x$ eigenstates $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

basis vector in one scheme is linear combo of both basis vectors in other

either basis a valid description of $\nu$ state
physical situation selects most natural choice:
• $\nu$ production/detection: Weak interaction $\rightarrow$ \textit{flavor} basis
• $\nu$ propagation in vacuum $\rightarrow$ \textit{mass} basis

basis vectors related by linear transformation

(P)MNS=Pontecorvo, Maki, Nakagawa, Sakata matrix

\begin{align*}
|\nu_{\text{flavor}}\rangle_{i \in e,\mu,\tau} &= \sum_{j=1,2,3} U_{ij} |\nu_{\text{mass}}\rangle_{j} \\
|\nu_{\text{mass}}\rangle_{i \in 1,2,3} &= \sum_{j=e,\mu,\tau} U^{\dagger}_{ij} |\nu_{\text{flavor}}\rangle_{j}
\end{align*} \tag{1}

$U$ is time-indep, unitary: $U^{-1} = U^{\dagger}$; $U^{\dagger}U = UU^{\dagger} = 1$
Neutrino Flavor Change

Key idea:
- neutrinos born in Weak interactions → created as Weak eigenstates
- propagate as vacuum eigenstates
- then detected in Weak interactions

*Evolution* of wavefunction during propagation changes probability of remaining a $\nu_e$ state

If mass eigenstates have definite $p$ and thus $E_j = \sqrt{p^2 + m_j^2}$ (as in vacuum), then Schrödinger:

$$i\hbar \frac{d}{dt} |\nu_{\text{mass}}\rangle_j = H_{\text{vacuum}} |\nu_{\text{mass}}\rangle_j = E_j |\nu_{\text{mass}}\rangle_j$$  \hspace{1cm} (3)

and so

$$|\nu_{\text{mass}}(t)\rangle_j = e^{-iE_j t/\hbar} |\nu_{\text{mass}}(0)\rangle_j$$  \hspace{1cm} (4)
Two flavors: allow 2 flavors (e and x) to mix
write \(|f\rangle = U_{\text{vac}}|m\rangle\), where
\[
U_{\text{V}} = \begin{pmatrix}
\cos \theta_{\text{V}} & \sin \theta_{\text{V}} \\
-\sin \theta_{\text{V}} & \cos \theta_{\text{V}}
\end{pmatrix}
\] (5)
with vacuum mixing angle \(\theta_{\text{V}} \in (0, \pi/4)\) ("\(\nu_e\) mostly \(\nu_1\)"")
\[
|\nu_e(t)\rangle = e^{-iE_1 t/\hbar} \cos \theta_{\text{V}} |1\rangle + e^{-iE_2 t/\hbar} \sin \theta_{\text{V}} |2\rangle
\] (6)
where \(E_1, E_2\) have same momentum \(p\)

Solar neutrinos start \((t = 0)\) as pure \(\nu_e\)
QM amplitude at \(t\) to remain \(\nu_e\):
\[
\langle \nu_e(0)|\nu_e(t)\rangle = e^{-iE_1 t/\hbar} \cos \theta_{\text{V}}^2 + e^{-iE_2 t/\hbar} \sin \theta_{\text{V}}^2
\] (7)
\(\Rightarrow\) probability to remain \(\nu_e\):
\[
|\langle \nu_e(0)|\nu_e(t)\rangle|^2 = 1 - \sin^2 2 \theta_{\text{V}} \sin^2 \left[ 1/2 \frac{(E_2 - E_1)t}{\hbar} \right]
\]
Since $m(\nu_i) \ll p$, $E_j = \sqrt{p^2 + m_j^2} \simeq p^2 + m_j^2 / 2p$, and

$$E_2 - E_1 \simeq \frac{m_2^2 - m_1^2}{2E} = \frac{\pm \Delta m^2}{2E}$$

(8)

$$\Delta m^2 = |m_2^2 - m_1^2| > 0$$

$E = \text{avg energy.}$

In time $t$ go distance $L \simeq ct$

$$P(\nu_e^{\text{birth}} \rightarrow \nu_e^{\text{detect}}) = |\langle \nu_e(0)|\nu_e(t)\rangle|^2$$

$$= 1 - \sin^2 2\theta \sin^2 \left( \pi \frac{L}{L_\nu} \right)$$

(9)

$$= 1 - \sin^2 2\theta \sin^2 \left[ 1.27 \frac{\Delta m^2 (\text{eV}^2) L (\text{km})}{E (\text{GeV})} \right]$$

where $L_\nu = 4\pi \hbar E / \Delta m^2$ “vacuum osc. length”
\[ P(\nu_e^{\text{birth}} \rightarrow \nu_e^{\text{detect}}) = |\langle \nu_e(0) | \nu_e(t) \rangle|^2 = 1 - \sin^2 2\theta_V \sin^2 \left( \frac{\pi L}{L_V} \right)\]

Minimum mass sensitivity: \( \pi L/L_V = \pi/2 \)
If \( L_V \ll 1 \text{ AU} \): wash out differences among species
If \( L_V \sim 1 \text{ AU} \): solve solar \( \nu \) problem!

\[ \Delta m^2 \sim 10^{-12} \text{ eV}^2 \left( \frac{E}{10 \text{ MeV}} \right) \] (10)
solves solar \( \nu \) problem, but dubious

Q: why?
⇒ “just-so” solution

also note: if \( \Delta m^2 \) larger, \( L_V \ll 1\text{AU} \)

\[ \Rightarrow |\langle \nu_e(0) | \nu_e(t) \rangle|^2 \simeq 1 - \frac{1}{2} \sin^2 2\theta \geq \frac{1}{2} \] (11)

but we need suppression \( > 50\% \)!
can’t do this with vacuum oscillations!