Astro 596/496 NPA
Lecture 27
Oct. 26, 2009

Announcements:

- Preflight 5 posted, due noon Friday

Last time: Solar Neutrino Problems and Solution

Q: what are the problems?

Q: what are the two main classes of solution? (pre-SNO)

Q: how does SNO show the nature of the solution?

Q: what does SNO imply for neutrino physics?

Solar Neutrino Problem(s) Pre-SNO

observed ν fluxes *less than* *Standard Solar Model* predictions

- Radiochemical: Chlorine, Gallium
- Water Čerenkov: Super-Kamiokande

  but $\nu_{\mathrm{super-k}}$ point back to Sun, have expected energy spectrum

Possible Solutions

- Standard Solar Model wrong–ν flux overpredicted (but pp?)
- Standard Model of particle physics wrong

Experimentum Crucis: SNO

- independently measure ^{8}B ν_e flux, all-flavor flux
- $\Phi_{\nu_{\mathrm{e}}}/\Phi_{\mathrm{tot}} = 0.31$

$\Rightarrow$ large non-ν_e flux arriving in detectors!

Implications: New Neutrino Physics!

The Sun makes only ν_e

Q: why? e.g., why not ν_μ*?*

$\longrightarrow$ if no new ν physics, only ν_e at Earth

$\longrightarrow$ predict $\Phi_{\rm CC}(\nu_e) = \Phi_{\rm NC}(\nu_x)$

SNO measures $\Phi_{\rm NC}(\nu_x) > \Phi_{\rm CC}(\nu_e)$!

with *very* high confidence!

non-ν_e flux arriving in detector!

A big deal:

- demands new neutrino physics
- indep. of detailed solar model

Triumph of the Standard Solar Model

SNO bonus: can infer total $^{8}\mathrm{B}$ ν flux

compare Bahcall SSM (Bahcall & Pinsonneault 2004):

$$\begin{aligned}\Phi_{\mathrm{SSM}}(^{8}\mathrm{B}) &= 5.79(1 \pm 0.23) \times 10^{6}\ \nu\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1} \\ &= [0.88 \pm 0.04(\mathrm{exp}) \pm 0.23(\mathrm{thy})]\, \Phi_{\mathrm{NC}}^{\mathrm{SNO}}\end{aligned}$$

consistent! SSM working extremely well!

⇒ major triumph for stellar evolution!

woo hoo!

2002 Nobel Prize in Physics: Ray Davis

Interlude: Updike Poem

Solar Neutrino Schizophrenia

total $\nu_e + \nu_\mu + \nu_\tau$ flux *in detectors*
agrees with SSM flux *out of solar core*

but solar νs must *start as ν_e*
→ neutrinos must **transmute** on the way!
i.e., $\nu_e \rightarrow \nu_{\mu,\tau}$!

there's more:

ν_e Experiment	$E_{\nu,\text{min}}$ Threshold	Obs/SSM
Gallium	> 0.233 MeV	$0.59 \pm 0.06 \pm 0.04$
Chlorine	> 0.814 MeV	$0.33 \pm 0.03 \pm 0.05$
Super-K	> 5 MeV	~ 0.4

⇒ transmutations must be energy-dependent:
Q: what should dependence be like?
`www: solar nu spectrum`

Solar Neutrino Transformation Properties

Need:

- small ν_e suppression at low energies (pp: $\lesssim$ 0.4 MeV)
- large ν_e suppression ($>$ 50%) at higher energies

Non-trivial neutrino physics required!

Neutrino Oscillations in Vacuum: The Quantum Neutrino

If *neutrinos have nonzero mass*

- family status (e, μ, τ "**flavor**"), and
- **mass**

can be **distinct**!

ν family → lepton number conservation in Weak interactions

formally, νs couple to Weak interaction as

flavor eigenstates

flavor basis vectors $|\nu_\alpha\rangle$, $\alpha = e, \mu, \tau$

free (vacuum) neutrino → *propagates* as

mass eigenstate

mass basis vectors $|j\rangle$, $j = 1, 2, 3$

Basis Transformation: Flavor/Weak $\leftrightarrow$ Mass/Vacuum

Key idea: **mass eigenstate $\neq$ flavor eigenstate**

analogous to spin-$\frac{1}{2}$: S_z eigenstates $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ vs S_x eigenstates $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

basis vector in one scheme is linear combo of *both* basis vectors in other

either basis a valid description of ν state

physical situation selects most natural choice:

- ν production/detection: Weak interaction → *flavor* basis
- ν propagation in vacuum → *mass* basis

basis vectors related by linear transformation

(P)MNS=Pontecorvo, Maki, Nakagawa, Sakata matrix

$$|\nu_{\text{flavor}}\rangle_{i\in e,\mu,\tau} = \sum_{j=1,2,3} U_{ij} |\nu_{\text{mass}}\rangle_j \qquad (1)$$

$$|\nu_{\text{mass}}\rangle_{i\in 1,2,3} = \sum_{j=e,\mu,\tau} U^\dagger_{ij} |\nu_{\text{flavor}}\rangle_j \qquad (2)$$

U is time-indep, unitary: $U^{-1} = U^\dagger$; $U^\dagger U = U U^\dagger = 1$

Neutrino Flavor Change

Key idea:

- neutrinos *born* in Weak interactions
  → *created* as *Weak* eigenstates
- *propagate* as *vacuum* eigenstates
- then *detected* in *Weak* interactions

Evolution of wavefunction during propagation
changes probability of remaining a ν_e state

If mass eigenstates have definite p and thus $E_j = \sqrt{p^2 + m_j^2}$ (as in vaccum), then Schrödinger:

$$i\hbar \frac{d}{dt} |\nu_{\mathrm{mass}}\rangle_j = H_{\mathrm{vacuum}} |\nu_{\mathrm{mass}}\rangle_j = E_j |\nu_{\mathrm{mass}}\rangle_j \tag{3}$$

and so

$$|\nu_{\mathrm{mass}}(t)\rangle_j = e^{-iE_j t/\hbar}\ |\nu_{\mathrm{mass}}(0)\rangle_j \tag{4}$$

Two flavors: allow 2 flavors (e and x) to mix
write $|f\rangle = U_{\mathrm{vac}}|m\rangle$, where

$$U_{\mathrm{V}} = \begin{pmatrix} \cos\theta_{\mathrm{V}} & \sin\theta_{\mathrm{V}} \\ -\sin\theta_{\mathrm{V}} & \cos\theta_{\mathrm{V}} \end{pmatrix} \tag{5}$$

with vacuum mixing angle $\theta_{\mathrm{V}} \in (0, \pi/4)$ ("ν_e mostly ν_1")

$$|\nu_e(t)\rangle = e^{-iE_1 t/\hbar} \cos\theta_{\mathrm{V}}|1\rangle + e^{-iE_2 t/\hbar} \sin\theta_{\mathrm{V}}|2\rangle \tag{6}$$

where E_1, E_2 have same momentum p

Solar neutrinos start ($t = 0$) as pure ν_e
QM **amplitude** at t to *remain* ν_e:

$$\langle\nu_e(0)|\nu_e(t)\rangle = e^{-iE_1 t/\hbar} \cos\theta_{\mathrm{V}}^2 + e^{-iE_2 t/\hbar} \sin\theta_{\mathrm{V}}^2 \tag{7}$$

⇒ probability to remain ν_e:

$$|\langle\nu_e(0)|\nu_e(t)\rangle|^2 = 1 - \sin^2 2\theta_{\mathrm{V}}\ \sin^2\left[1/2\ \frac{(E_2 - E_1)t}{\hbar}\right]$$

Since $m(\nu_i) \ll p$, $E_j = \sqrt{p^2 + m_j^2} \simeq p^2 + m_j^2/2p$, and

$$E_2 - E_1 \simeq \frac{m_2^2 - m_1^2}{2E} = \frac{\pm\Delta m^2}{2E} \tag{8}$$

$\Delta m^2 = |m_2^2 - m_1^2| > 0$

E = avg energy.

In time t go distance $L \simeq ct$

$$\begin{aligned} P(\nu_e^{\text{birth}} \rightarrow \nu_e^{\text{detect}}) &= |\langle \nu_e(0)|\nu_e(t)\rangle|^2 \\ &= 1 - \sin^2 2\theta_{\text{V}} \sin^2\left(\pi\frac{L}{L_{\text{V}}}\right) \qquad (9) \\ &= 1 - \sin^2 2\theta_{\text{V}} \sin^2\left[1.27\frac{\Delta m^2(\text{eV}^2)L(\text{km})}{E(\text{GeV})}\right] \end{aligned}$$

where $L_{\text{V}} = 4\pi\hbar E/\Delta m^2$ "vacuum osc. length"

$$P(\nu_e^{\text{birth}} \rightarrow \nu_e^{\text{detect}}) = |\langle \nu_e(0)|\nu_e(t)\rangle|^2 = 1 - \sin^2 2\theta_{\text{V}} \ \sin^2\left(\frac{\pi L}{L_{\text{V}}}\right)$$

Minimum mass sensitivity: $\pi L/L_{\text{V}} = \pi/2$
If $L_{\text{V}} \ll 1$ AU: wash out differences among species
If $L_{\text{V}} \simeq 1$ AU: solve solar ν problem!

$$\Delta m^2 \sim 10^{-12} \ \text{eV}^2 \ \left(\frac{E}{10 \ \text{MeV}}\right) \tag{10}$$

solves solar ν problem, but dubious
Q: why?
⇒ "just-so" solution

also note: if Δm^2 larger, $L_V \ll 1AU$

$$\Rightarrow \quad |\langle \nu_e(0)|\nu_e(t)\rangle|^2 \simeq 1 - \frac{1}{2}\sin^2 2\theta \geq \frac{1}{2} \tag{11}$$

but we need suppression $> 50\%$!
can't do this with vaccuum oscillations!