Astro 596/496 NPA
Lecture 21
Oct. 12, 2009

Announcements:

- Preflight 4 due noon Friday
- thanks to all Fermions for a great trip!

Last time: began particle dark matter
consider stable/long-lived particle χ

- *equal* numbers of $\bar{\chi}$, or $\chi = \bar{\chi}$
- nonrelativistic today: $m_\chi \gg T_0 \sim 3 \times 10^{-4}$ meV
- if *still in (chemical) equilib.*: $n_\chi \sim e^{-m_\chi/T} \to 0$ today
  i.e., all $\chi\bar{\chi}$ have annihilated away

$\Rightarrow$ if χ is DM: must have dropped *out of equilibrium*

Particle Dark Matter: Thermal Relics

Kolb & Turner, Ch. 5; Dodelson Ch. 3.4

if nonbaryonic stable particles created in Early U.
$\longrightarrow$ dark matter today (?)

consider an exotic particle ψ which is:

- *stable* (doesn't) decay
- created in early U., along with $\bar{\psi}$
- can *annihilate* via $\psi\bar{\psi} \leftrightarrow X\bar{X}$
  note: could be that $\psi = \bar{\psi} \rightarrow$ annihilations unavoidable!

if $\psi\bar{\psi}$ made in pairs (or $\psi = \bar{\psi}$)
$\longrightarrow$ thermally produce $n_\psi = n_{\bar{\psi}}$ in early U.
if only *annihilate* afterwards: $n_\psi = n_{\bar{\psi}}$ *always*
Q: if these are DM, why don't they just annihilate today?

Freezeout and Relic Abundance of a Symmetric Species

for *conserved* species ψ (chem. pot. $\mu_\psi \neq 0$)

constant comoving number: $d(na^3) \overset{\text{cons}}{=} 0$

$$\Rightarrow \boxed{\dot{n}_\psi + 3\frac{\dot{a}}{a}\, n_\psi \overset{\text{cons}}{=} 0}$$

for *non-conserved* species: $d(n_\psi a^3) = qa^3\ dt \neq 0$, where
q = *source/sink* rate = creation/destruction rate per unit vol

$$\Rightarrow \boxed{\dot{n}_\psi + 3\frac{\dot{a}}{a}\, n_\psi = q}$$

assume annihilation $\psi\bar{\psi} \to X\bar{X}$ product X thermal,
with chem. pot. $\mu_X \ll T \;\Rightarrow\; n_X = n_{\bar{X}}$

$$\begin{aligned} q = q_{\text{net}} &= q_{\text{prod}} - q_{\text{ann}} && (1) \\ &= \langle \sigma v \rangle_{\text{prod}} n_X n_{\bar{X}} - \langle \sigma v \rangle_{\text{ann}} n_\psi n_{\bar{\psi}} && (2) \\ &= \langle \sigma v \rangle_{\text{prod}} n_X^2 - \langle \sigma v \rangle_{\text{ann}} n_\psi^2 && (3) \end{aligned}$$

in equilib, *Q: what condition holds for q?*

chem equilib: $q = 0 \Rightarrow \boxed{q_{\text{prod}} = q_{\text{ann}}}$
so in general

$$\dot{n}_\psi + 3Hn_\psi = q = -\langle\sigma v\rangle_{\text{ann}} \left[n_\psi^2 - (n_\psi^{\text{eq}})^2\right] \tag{4}$$

and a similar expression for $\bar{\psi}$

Change variables:
(1) use comoving coords:
  photon density $n_\gamma \propto T^3 \propto a^{-3}$,
  so put $Y = n_\psi / n_\gamma$ to remove volume dilution
  then $\dot{n_\psi} + 3\dot{a}/a\ n_\psi = n_\gamma \dot{Y}$
(2) put $x = m_\psi / T \propto a$
  since $t \propto 1/T^2 \propto x^2$,
  $dY/dt = dY/dx\ \dot{x} = H\ x\ dY/dx$

Then:

$$Hx\frac{dY}{dx} = -n_\gamma \langle\sigma v\rangle_{\text{ann}} \left(Y^2 - Y_{\text{eq}}^2\right) \tag{5}$$

$$\tag{6}$$

finally

$$\frac{x}{Y_{\text{eq}}}\frac{dY}{dx} = -\frac{\Gamma_A}{H}\left[\left(\frac{Y}{Y_{\text{eq}}}\right)^2 - 1\right] \tag{7}$$

where $\Gamma_A = n_\psi^{\text{eq}} \langle \sigma v \rangle_{\text{ann}}$: annihil. rate

So: change in comoving ψ controlled by
(1) annihil. effectiveness Γ/H
(2) deviation from equil

when $\Gamma/H \gg 1$
Q: what if $Y < Y_{\text{eq}}$? $Y > Y_{\text{eq}}$?

when $\Gamma/H < 1$
Q: how does Y change?

Q: how you you expect Y to evolve?

when $\Gamma/H \gg 1$, Y driven to Y_{eq}
when $\Gamma/H < 1$, Y change is small $\rightarrow$ freezeout!

relic abundance at $T \rightarrow 0$ or $x \rightarrow \infty$ is
$Y_\infty \simeq Y_{\text{eq}}(x_f)$: value at freezeout

Step back:
How can a symmetric species have
$n_\psi = n_{\bar{\psi}} \neq 0$ at $T \ll m$?

physically: expansion is key

if $H = 0$, $Y_\infty = Y_{\rm eq}(\infty) = 0$:

$\rightarrow$ all ψ find $\bar{\psi}$ partner, annihilate

but $H \neq 0$: when U dilute enough,

ψ never finds $\bar{\psi}$: i.e., $\Gamma \ll H$

nonzero relic abundance

hot relics: $x_f \ll 1$ ($T_f \gg m$)

cold relics: $x_f \gg 1$

Note: hot/cold *relics* refers to freezeout conditions

But: hot/cold *dark matter* refers to structure formation criteria

(namely, m vs temp $T_{\rm eq}$ at matter-rad equality)

Hot Relics: Neutrinos

Neutrinos are an obvious and attractive candidate
for non-baryonic dark matter *Q: because?*

⋆ Laboratory studies of β decay
e.g., precision measurement of e^- energy in $^3\text{H} \rightarrow {^3\text{He}} + e^- + \bar{\nu}_e$
place limits on ν_e mass *Q: how?*
current PDG imit: $\boxed{m(\nu_e) < 2 \text{ eV}}$
⋆ We shall see: *solar* and *atmospheric* νs
will ultimately show *all* 3 species have $\boxed{m(\nu) \lesssim few \text{ eV}}$
$\Rightarrow$ so neutrinos are definitely **hot relics** *Q: becuase?*

But you will show (PS 4): neutrino density parameter is

$$\Omega_\nu \simeq \frac{\sum_i m(\nu_i)}{45 \text{ eV}} \tag{8}$$

Q: implications for dark matter?
Q: implications for particle physics?

Dark Matter Requires New Physics

no viable particle dark matter candidates
in Standard Model of particle physics

non-baryonic DM **demands** *physics beyond the Standard Model*

Hugely important and exciting for particle physics!

Unlike dark energy: particle physics *does* offer solutions!
particle candidates available "off the shelf"
invented for particle physics motivations independent from DM!
- lightest supersymmetric particle
- axion
- strangelets...

(Almost) all of these are formed as *cold relics*

Cold Relics: WIMPs

cold relic: non-relativistic at freezeout

so $x_f = m/T_f \gg 1 \to T_f \ll m$

$\Rightarrow n_{\rm eq} \sim e^{-m/T}(mT)^{3/2}$

$\Rightarrow Y_{\rm eq} \sim e^{-x}x^{3/2}$

Freezeout:

$\Gamma_{\rm ann} = H$ at $T = T_f$

$\Rightarrow n_{\rm eq}\langle\sigma v\rangle_{\rm ann} \sim \sqrt{G}T^2$

what needed to find value of T_f?

To solve:

- need annihilation cross section
  for many models, $\langle \sigma v \rangle \propto v^n$ (S-wave: $n = 0$)
  $\Rightarrow \langle \sigma v \rangle(x) = \sigma_1 c x^{n/2}$, where $\sigma_1 = \sigma(E = m)$
- convenient rewrite $1/\sqrt{G} = M_{\mathrm{Pl}} \simeq 10^{19}$ GeV
  (Planck Mass)

set $\Gamma_{\mathrm{ann}}(T_f) = H(T_f)$, and solve for T_f

Find: $x_f \sim \ln(m M_{\mathrm{Pl}} \sigma_1) \Rightarrow T_f = m/x_f$

So

$$Y_\infty \simeq Y_{\mathrm{eq}}(x_f) \tag{9}$$

$$\sim \frac{x_f^{3/2}}{m M_{\mathrm{Pl}} \sigma_1} \tag{10}$$

→ present relic number density

$$n_{\psi,0} = Y_\infty n_{\gamma,0} = 400\ Y_\infty\ \mathrm{cm}^{-3} \tag{11}$$

present relic mass density

$$\rho_{\psi,0} = m n_{\psi,0} \simeq \frac{x_f^{3/2} n_{\gamma,0}}{M_{\mathrm{Pl}}\sigma_1} \tag{12}$$

What have we shown?
if a symmetric stable species ever created
(annihilates but not decays)
then annihilations will freeze, and
inevitably have nonzero relic density today.

This calculation is of the highest interest to particle physicists
Q: why?
We have calculated a relic density
Q: To what should this be compared?

Cold Relics: Present Abundance

⋆ $\rho_{\psi,0}$ indep of m_ψ

⋆ $\rho_{\psi,0} \propto 1/\sigma_1$: the weak prevail!
Q: what sort of cross section is relevant here?

⋆ To get "interesting" present density:
$\Omega_\psi \sim 1 \rightarrow \rho_\psi \sim \rho_{\text{crit}}$ *demands* a *specific* cross section

$$\sigma_1 = \frac{x_f^{3/2} n_{\gamma,0}}{\Omega_\psi M_{\text{p}} \rho_{\text{crit}}} \quad (13)$$

$$\sim 10^{-37}\ \text{cm}^2 \left(\frac{x_f}{10}\right)^{3/2} \quad (14)$$

scale of the Weak interaction! $[\sigma_{\text{weak}}(E \sim \text{GeV}) \sim 10^{-38}\ \text{cm}^2]$

The WIMP Miracle

Dark Matter candidate:
if DM is a cold symmetric relic
needed *annihilation cross section* is at Weak scale!
corresponding energy: if $\sigma \sim \alpha/E^2$
then $\sigma \sim 10^{-36}$ cm$^2 = 10$ pb $\rightarrow E \sim 1$ TeV

deeper reason for DM as
Weakly Interacting Massive Particle: **WIMP**

that weak-scale annihilations $\rightarrow \Omega_\chi \sim \Omega_{\rm nbdm}$: **"WIMP Miracle"**

How to find them?
What if we do? What if we don't?