Astro 596/496 NPA
Lecture 24
Oct. 19, 2009

Announcements:

- Problem Set 4 due next time

Last time: finished cosmology

Taking Stock:

`www: Cosmic Pie Chart`

Q: where do we think the slices come from?

`www: Solar Abundances`

Q: what features do we now understand? what remains?

Solar Models

Charity begins at home:

understand the Sun first

focus on solar neutrinos:

new physics, powerful diagnostic of solar/stellar models

Basic Assumptions? Ingredients?

Solar Model Ingredients (ASTR 404, 504)

1. **Hydrostatic equilibrium**: pressure–gravity balance
consider spherical shell of width dr,
vol $dV = 4\pi r^2 dr$
net weight: $mg = \rho dV \; Gm(r)/r^2 = 4\pi Gm(r)\rho dr$
pressure diff: $P_{\rm net} = -P(r+dr) + P(r) = -dP/dr \; dr$
$\Rightarrow$ force: $F_p = P_{\rm net} A = -4\pi r^2 P_{\rm net}$ (up)
balance:

$$-\frac{dP}{dr} = \frac{Gm(r)\rho}{r^2} \tag{1}$$

using $dm(r)/dr = 4\pi r^2 \rho(r)$ (Lagrangian "mass coordinate")

$$-\frac{dP}{dm} = \frac{Gm\,dm/dr}{4\pi r^4} \tag{2}$$

Equation of State: $p = \rho kT/m + aT^4/3$

2. **Energy conservation and transport:**

Center: radiation

Envelope: convection (recall ^{7}Li depletion)

energy loss is via photons & is diffusive.

energy flux is

$$F = \langle c p_r v_r n \rangle = \frac{ca}{3} T^4 \tag{3}$$

(where $\rho_{\rm rad} c^2 = a T^4$)

net flux at r: $F_{\rm net} = F(r + \delta r) - F(r) \simeq dF/dr\ \delta r$

diffusion: "stepsize" δr is mfp $\lambda = 1/n\sigma \equiv 1/\rho\kappa$

opacity $\kappa = \sigma n/\rho = \sigma/m$

local luminosity: $\ell = 4\pi r^2 F_{\rm net}$

$$\frac{\ell}{4\pi r^2} = \frac{1}{\rho\kappa}\frac{dF}{dr} = \frac{4acT^3}{3\rho\kappa}\frac{dT}{dr} \tag{4}$$

3. **Energy generation via nuke reactions**

put $\rho\varepsilon$ = nuke energy production rate per unit vol

$$d\ell = \rho\varepsilon dV = \rho\varepsilon 4\pi r^2 \, dr \quad (5)$$

$$\frac{d\ell}{dr} = 4\pi r^2 \rho\varepsilon \quad (6)$$

if $q = \langle \sigma_{ab} v \rangle n_a n_b$

= nuke reaction rate per vol for $a + b \rightarrow c + d$

$\rho\varepsilon = Qq$, where energy release $Q = \Delta_a + \Delta_b - \Delta_c - \Delta_d$

Now have differential equations

but still need one more thing to solve them

What's that?

4. Boundary conditions:

$t_\odot = 4.6$ Gyr

$M_{\text{tot}} = M_\odot = 2.0 \times 10^{33}$ g

$R = R(t_\odot) = R_\odot = 7.0 \times 10^{10}$ cm

$L = L_\odot = 3.8 \times 10^{33}$ erg/s

With these, solve $m(r)$, $\ell(m)$, $T(m)$ (vs time)

for nuke rxns, we will need central ρ_c, T_c

Back of the Envelope

Order of magnitude:

$$\frac{dP}{dR} \sim \frac{P_c}{R} \tag{7}$$

$$\sim \frac{GM\rho}{R^2} \tag{8}$$

ideal gas: $P = \rho k T/m$

$$T_c \sim \frac{(m_p/2)P_c}{\rho k} = \frac{GMm_p}{2kR} \sim 10^7 \text{ K} \tag{9}$$

Why $m_p/2$*?*

now compare to professional result...

Standard Solar Model (SSM)
Bahcall & Pinsonneault (2000,2004)

conditions at solar center:

$$T_c = 1.57 \times 10^7 \text{ K} \quad (10)$$

$$\rho_c = 152 \text{ g cm}^{-3} \quad (11)$$

$$X_c = \left(\frac{\rho_{\rm H}}{\rho_{\rm B}}\right)_c = 0.34 \quad (12)$$

$$Y_c = \left(\frac{\rho_{\rm He}}{\rho_{\rm B}}\right)_c = 0.64 \quad (13)$$

Sun: main sequence, $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e$

∞ *Q: reaction steps?*

Solar Hydrogen Burning: Big Picture

Sun: main sequence, $4p \rightarrow {}^4\mathrm{He} + 2e^+ + 2\nu_e$

Reaction chains usually begin with

$$p + p \rightarrow d + e^+ + \nu_e \tag{14}$$

weak rxn: slow; then

$$d + p \rightarrow {}^3\mathrm{He} + \gamma \tag{15}$$

Then: 3 branches

PP-I

$${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + p + p$$

$${}^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$$

PP-II

$${}^7\mathrm{Be} + e \rightarrow {}^7\mathrm{Li} + \nu_e$$

$${}^7\mathrm{Li} + p \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$$

PP-III

$${}^7\mathrm{Be} + p \rightarrow {}^8\mathrm{B} + \gamma$$

$${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be} + e^+ + \nu_e$$

$${}^8\mathrm{Be} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$$

The PP-I Chain

Deuterium

d source $pp \to d e^+ \nu_e$, rate per vol $\lambda_{pp} n_p^2/2$
d sink: $dp \gtrsim\ ^3\text{He}\gamma$, rate $\lambda_{dp} n_d n_p$

evolution of d:

$$\begin{aligned} \dot{n}_d &= -\lambda_{dp} n_d n_p + \lambda_{pp} n_p^2/2 && (16) \\ &= -\Gamma_{\text{per}\, d}(dp \to {}^3\text{He}\gamma) \left(n_d - n_d^{\text{eq}} \right) && (17) \end{aligned}$$

where $\Gamma_{\text{per}\, d} = n_p \langle \sigma v \rangle_{dp \to {}^3\text{He}\gamma} \equiv n_p \lambda_{dp}$

self-regulating:
driven to equilibrium $\dot{n}_d = 0$
in timescale $\tau_d = 1/\Gamma_{\text{per}\, d} \sim 1$ s (!)

$$\left(\frac{\text{D}}{\text{H}} \right)_{\text{eq}} \sim 10^{-18} \quad (18)$$

Helium-3

source: $dp \rightarrow {}^3\text{He}\gamma$

dominant sink: ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + p + p$

$$\dot{n}_3 = -2\lambda_{33} n_3^2/2 + \lambda_{dp} n_d n_p \tag{19}$$

approx: $n_d = n_d^{\text{eq}}$

at equil.,

$$n_3^{\text{eq}} = \sqrt{\frac{\lambda_{dp}}{2\lambda_{33}} n_d^{\text{eq}} n_p} = \sqrt{\frac{\lambda_{dp}}{2\lambda_p}} n_p \tag{20}$$

so in solar core

$$\left(\frac{{}^3\text{He}}{\text{H}}\right)_{\text{eq}} \sim 10^{-5} \tag{21}$$

reached in timescale $\tau_3 \sim 10^6$ yr

...and longer at lower temp

$\Rightarrow$ large ^{3}He gradient in the Sun

Helium-4

in PP-I, source is $^3\mathrm{He}\,^3\mathrm{He} \rightarrow \,^4\mathrm{He}\,pp$

no sink

Q: so what is equilib abundance?

$$\dot{n}_4 = \lambda_3 3 n_3^2/2 = \lambda_{33}(n_3^{\mathrm{eq}})^2 = \lambda_{pp} n_p^2/4 \qquad (22)$$

The pp-II, pp-III Chains

Other main pp chains: different ^{3}He fate

^{7}Be branching key:

e capture rate $\sim 1000\times$ p capture rate

- ^{7}Be: 15% of ν production
- ^{8}B $\sim 0.02\%$ of ν production

The CNO Cycle

pre-existing C, N, O act as $4p \rightarrow {}^4$He *catalyst*

$$\begin{array}{ccccc}
{}^{12}\mathrm{C} & \overset{(p,\gamma)}{\rightarrow} & {}^{13}\mathrm{N} & \overset{e^+\nu_e}{\rightarrow} & {}^{13}\mathrm{C} \\
(p,\alpha)\uparrow & & & & \downarrow(p,\gamma) \\
{}^{15}\mathrm{N} & \overset{e^+\nu_e}{\leftarrow} & {}^{15}\mathrm{O} & \overset{(p,\gamma)}{\leftarrow} & {}^{14}\mathrm{N}
\end{array}$$

Coulomb barriers high ($Z = 6, 7, 8$): *need high T_c* to go

$\Rightarrow$ CNO cycle minor in Sun (CNO $\rightarrow$1.6% $L_\odot$)

but main H-burner for $M \gtrsim 1.5 M_\odot$