

Astro 596/496 NPA

Lecture 24

Oct. 19, 2009

Announcements:

- Problem Set 4 due next time

Last time: finished cosmology

Taking Stock:

www: Cosmic Pie Chart

*Q: where do we think the slices come from?*

www: Solar Abundances

└ *Q: what features do we now understand? what remains?*

## Solar Models

Charity begins at home:

- understand the Sun first

- focus on solar neutrinos:

- new physics, powerful diagnostic of solar/stellar models

*Basic Assumptions? Ingredients?*

# Solar Model Ingredients (ASTR 404, 504)

1. **Hydrostatic equilibrium**: pressure–gravity balance

consider spherical shell of width  $dr$ ,

$$\text{vol } dV = 4\pi r^2 dr$$

$$\text{net weight: } mg = \rho dV \frac{Gm(r)}{r^2} = 4\pi Gm(r)\rho dr$$

$$\text{pressure diff: } P_{\text{net}} = -P(r + dr) + P(r) = -dP/dr \, dr$$

$$\Rightarrow \text{force: } F_p = P_{\text{net}}A = -4\pi r^2 P_{\text{net}} \text{ (up)}$$

balance:

$$-\frac{dP}{dr} = \frac{Gm(r)\rho}{r^2} \quad (1)$$

using  $dm(r)/dr = 4\pi r^2 \rho(r)$  (Lagrangian “mass coordinate”)

$$-\frac{dP}{dm} = \frac{Gm dm/dr}{4\pi r^4} \quad (2)$$

$\omega$

$$\text{Equation of State: } p = \rho kT/m + aT^4/3$$

## 2. Energy conservation and transport:

Center: radiation

Envelope: convection (recall  ${}^7\text{Li}$  depletion)

energy loss is via photons & is diffusive.

energy flux is

$$F = \langle cp_r v_r n \rangle = \frac{ca}{3} T^4 \quad (3)$$

(where  $\rho_{\text{rad}} c^2 = aT^4$  )

net flux at  $r$ :  $F_{\text{net}} = F(r + \delta r) - F(r) \simeq dF/dr \delta r$

diffusion: “stepsize”  $\delta r$  is mfp  $\lambda = 1/n\sigma \equiv 1/\rho\kappa$

opacity  $\kappa = \sigma n/\rho = \sigma/m$

local luminosity:  $\ell = 4\pi r^2 F_{\text{net}}$

$$\frac{\ell}{4\pi r^2} = \frac{1}{\rho\kappa} \frac{dF}{dr} = \frac{4acT^3}{3\rho\kappa} \frac{dT}{dr} \quad (4)$$

### 3. Energy generation via nuke reactions

put  $\rho\varepsilon$  = nuke energy production **rate** per unit vol

$$d\ell = \rho\varepsilon dV = \rho\varepsilon 4\pi r^2 dr \quad (5)$$

$$\frac{d\ell}{dr} = 4\pi r^2 \rho\varepsilon \quad (6)$$

if  $q = \langle \sigma_{ab} v \rangle n_a n_b$

= nuke reaction rate per vol for  $a + b \rightarrow c + d$

$\rho\varepsilon = Qq$ , where energy release  $Q = \Delta_a + \Delta_b - \Delta_c - \Delta_d$

Now have differential equations

but still need one more thing to solve them

*What's that?*

4. Boundary conditions:

$$t_{\odot} = 4.6 \text{ Gyr}$$

$$M_{\text{tot}} = M_{\odot} = 2.0 \times 10^{33} \text{ g}$$

$$R = R(t_{\odot}) = R_{\odot} = 7.0 \times 10^{10} \text{ cm}$$

$$L = L_{\odot} = 3.8 \times 10^{33} \text{ erg/s}$$

With these, solve  $m(r)$ ,  $\ell(m)$ ,  $T(m)$  (vs time)  
for nuke rxns, we will need central  $\rho_c$ ,  $T_c$

## Back of the Envelope

Order of magnitude:

$$\frac{dP}{dR} \sim \frac{P_c}{R} \quad (7)$$

$$\sim \frac{GM\rho}{R^2} \quad (8)$$

ideal gas:  $P = \rho kT/m$

$$T_c \sim \frac{(m_p/2)P_c}{\rho k} = \frac{GMm_p}{2kR} \sim 10^7 \text{ K} \quad (9)$$

*Why  $m_p/2$ ?*

now compare to professional result...

Standard Solar Model (SSM)

Bahcall & Pinsonneault (2000,2004)

conditions at solar center:

$$T_c = 1.57 \times 10^7 \text{ K} \quad (10)$$

$$\rho_c = 152 \text{ g cm}^{-3} \quad (11)$$

$$X_c = \left( \frac{\rho_{\text{H}}}{\rho_{\text{B}}} \right)_c = 0.34 \quad (12)$$

$$Y_c = \left( \frac{\rho_{\text{He}}}{\rho_{\text{B}}} \right)_c = 0.64 \quad (13)$$

Sun: main sequence,  $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e$

$\infty$  Q: reaction steps?

## Solar Hydrogen Burning: Big Picture

Sun: main sequence,  $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e$

Reaction chains usually begin with

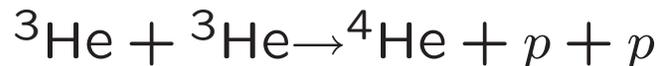


weak rxn: slow; then

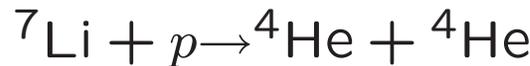
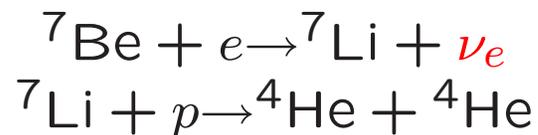


Then: 3 branches

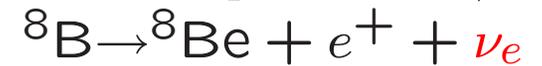
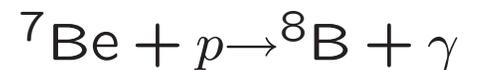
PP-I



PP-II



PP-III



## The PP-I Chain

### Deuterium

*d* source  $pp \rightarrow de^+\nu_e$ , rate per vol  $\lambda_{pp}n_p^2/2$

*d* sink:  $dp \rightarrow {}^3\text{He}\gamma$ , rate  $\lambda_{dp}n_d n_p$

evolution of *d*:

$$\dot{n}_d = -\lambda_{dp}n_d n_p + \lambda_{pp}n_p^2/2 \quad (16)$$

$$= -\Gamma_{\text{per } d}(dp \rightarrow {}^3\text{He}\gamma) (n_d - n_d^{\text{eq}}) \quad (17)$$

where  $\Gamma_{\text{per } d} = n_p \langle \sigma v \rangle_{dp \rightarrow {}^3\text{He}\gamma} \equiv n_p \lambda_{dp}$

self-regulating:

driven to equilibrium  $\dot{n}_d = 0$

in timescale  $\tau_d = 1/\Gamma_{\text{per } d} \sim 1 \text{ s (!)}$

$$\left(\frac{\text{D}}{\text{H}}\right)_{\text{eq}} \sim 10^{-18} \quad (18)$$

## Helium-3

source:  $dp \rightarrow {}^3\text{He}\gamma$

dominant sink:  ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + p + p$

$$\dot{n}_3 = -2\lambda_{33}n_3^2/2 + \lambda_{dp}n_d n_p \quad (19)$$

approx:  $n_d = n_d^{\text{eq}}$

at equil.,

$$n_3^{\text{eq}} = \sqrt{\frac{\lambda_{dp}}{2\lambda_{33}}n_d^{\text{eq}}n_p} = \sqrt{\frac{\lambda_{dp}}{2\lambda_p}n_p} \quad (20)$$

so in solar core

$$\left(\frac{{}^3\text{He}}{\text{H}}\right)_{\text{eq}} \sim 10^{-5} \quad (21)$$

reached in timescale  $\tau_3 \sim 10^6$  yr

⇕ ...and longer at lower temp

⇒ large  ${}^3\text{He}$  gradient in the Sun

## Helium-4

in PP-I, source is  ${}^3\text{He}{}^3\text{He} \rightarrow {}^4\text{He}pp$

no sink

*Q: so what is equilib abundance?*

$$\dot{n}_4 = \lambda_3 3n_3^2/2 = \lambda_{33}(n_3^{\text{eq}})^2 = \lambda_{pp}n_p^2/4 \quad (22)$$

## The $pp$ -II, $pp$ -III Chains

Other main  $pp$  chains: different  ${}^3\text{He}$  fate

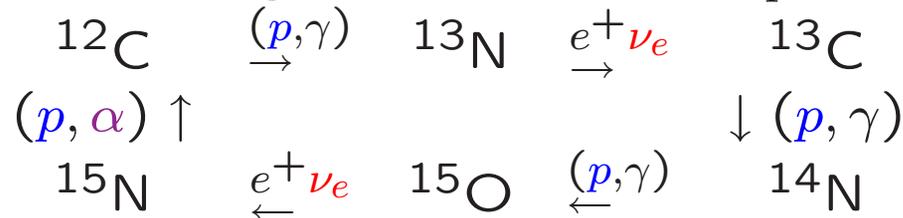
${}^7\text{Be}$  branching key:

$e$  capture rate  $\sim 1000\times$   $p$  capture rate

- ${}^7\text{Be}$ : 15% of  $\nu$  production
- ${}^8\text{B}$   $\sim 0.02\%$  of  $\nu$  production

## The CNO Cycle

*pre-existing C, N, O* act as  $4p \rightarrow {}^4\text{He}$  *catalyst*



Coulomb barriers high ( $Z = 6, 7, 8$ ): *need high  $T_c$*  to go

$\Rightarrow$  CNO cycle minor in Sun (CNO  $\rightarrow 1.6\% L_\odot$ )

but main H-burner for  $M \gtrsim 1.5M_\odot$