Astro 210

Lecture 40

December 6, 2010

Announcements

- Final Exam: next Monday Dec 13, 7–10 pm, here as usual
  `www: info online`
- **ICES** course evaluation available online
  please fill it out–I *do* read & use results
- *check Compass GradeBook scores for accuracy*

Now that your astro-muscles are strong

You *fully qualify* for upper-level astrophysics **courses**

- ASTR 404: Stellar Astrophysics
- ASTR 405: Solar System and Interstellar Medium
- ASTR 406: Galaxies and the Universe
- ASTR 411: Astronomical Techniques

If you are a LAS Physics major
it's cheap to add **Astronomy Major**
$\rightarrow$ many requirements are the same
after ASTR 210: 18 advanced hours, of which $\geq$ 10 from ASTR

If you are in Engineering, it's cheap to add an **Astronomy Minor**
after ASTR 210: 15 ASTR hours, $\geq$ 9 of which are advanced

iClicker Poll: ICES

Vote your conscience!

Did you get one or more ICES notification emails for this course?

A Yes, and I promise to fill out the form if I have not already

B No, but I promise to go to https://ices.cte.uiuc.edu/ and fill out the form if I have not already

Last time: Cosmology

- cosmic structure

  Q: cosmic "building blocks"? *Q: distribution in space?*
- cosmic dynamics
- *Q: what is observed? Hubble's law?*

galaxies are cosmic “building blocks”
like stars are basic unit of galaxies

Cosmic Structure

on large scales, Universe is

1. **homogeneous**:
   galaxies fill space with (nearly) uniform density
2. **isotropic**:
   universe looks same in all directions

Cosmic Dynamics

galaxies all show *redshifts*: in motion, away from us
Hubble (1929): farther $\rightarrow$ faster

Hubble sez: farther $\rightarrow$ faster

more specifically: Hubble Law

$$v = Hd \tag{1}$$

H: "Hubble constant" (sometimes written H_0)

Q: units of H?

value: $H_0 \approx 70$ km/(s $\cdot$ Mpc)

draw random (homog) dots

Q: add velocity vectors–what's the pattern?

What does it mean?

Q: logical possibilities?

The Meaning of Hubble's Law

observe: all galaxies move away from us!
galaxy motion *not* random, but rather highly organized
$\rightarrow$ cries out for interpretation!

Egoist view: we are at center of Universe!

but:
- MW typical galaxy, not special
- center un-Copernican

Einstein view:
no center! ...and no edge! Universe is homogeneous!

the universe is expanding

highfalutin mathematics:
Hubble's law a very special mathematical form
in fact, the **only** form that has the following property:
`www: expanding universe animation`

Hubble's law universal:
in expanding U, **all** galaxies
find $v = Hd$
$\rightarrow$ don't need to live in a special place to
find Hubble's law

Revolution Re-Re-Revisited

Copernican Revolution I (17th Century):
Earth is one typical planet among many
not center of solar system

Copernican Revolution II (earth 20th Century):
Sun is one typical star among many
not center of Milky Way Galaxy

Copernican Revolution III (1920's):
Milky Way is one typical galaxy among many
Universe much larger than previously thought

Copernican Revolution IV (1929):
we live in an expanding universe
which has no center at all!

Expanding Universe: Theory

recall General Relativity (Einstein Gravity):
★ space (& time) dynamic!
★ Universe is dramatic example of this

on cosmic lengthscales, space uniformly expands
stretches "like rubber sheet"
diagram: grid lines, galaxies at t_1, $t_2 > t_1$

Describe mathematically: write $d(t) = a(t)r$
$d(t)$: physical separation
r: fixed coords on rubber sheet
$a(t)$: measures expansion – "scale factor" or stretch-o-meter
put present age of Universe: $t = t_0$ today
then $a(t_0) = a_0 = 1$

Expansion Rate

Consider two observers (e.g., galaxies) at time t
separated by distance $d(t) = a(t)r$

what is velocity of one point as measured by the other?

$$v(t) = \frac{\partial}{\partial t} d = \frac{\partial}{\partial t}(ar) = \dot{a}r \tag{2}$$

$$= \frac{\dot{a}}{a}\, ar = \frac{\dot{a}}{a}\, d \equiv H\ d \tag{3}$$

so expansion gives $v = Hd$, with $H = \dot{a}/a$

★ Hubble's law!

★ we see $H = H(t)$ measures expansion rate
and need not be constant! → "Hubble Parameter"
H_0 is present value, can (& does!) change with time

How does the Universe expand?

- need to find $a(t) \to$ encodes cosmic expansion history
- How to find $a(t)$?

  similar to Newtonian physics we know and love:

  find law of motion that relates motion to gravity sources

consider two points (observers) in Universe at t

- (arbitrary) central point at $r = 0$
- a test mass m at distance $d(t) = a(t)\, r$

find energy, motion of test mass

$\to$ tells motion of any arbitrary point in universe

$\to$ cosmic dynamics!

what is energy, motion of test mass?

- sees spherical "enclosed" cosmic mass
  $M = \rho V_{\text{sphere}} = 4\pi/3 \; d^3 \rho$
- has energy $E_{\text{tot}} = KE + PE = const$ (< 0 if bound)

$$\frac{1}{2} m v^2 - \frac{GMm}{d} = const \tag{4}$$

$$m \left[\frac{1}{2} (Hd)^2 - \frac{4\pi G}{3} \rho d^2 \right] = const \tag{5}$$

$$H^2 - \frac{8\pi G}{3} \rho = \frac{2const}{md^2} = -\frac{\kappa}{a^2} \tag{6}$$

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{\kappa_2}{a} \tag{7}$$

Friedmann equation for evolution of $a(t)$

Cosmodynamics

Friedmann is cosmic "equation of motion"
kinda a cosmic "$F = ma$" (really, cosmic energy equation)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{\kappa}{a^2} \tag{8}$$

Which means that the evolution of the universe

- expansion rate $H = \dot{a}/a$, and thus
- cosmic history $a(t)$, are controlled by

are controlled by the contents of the universe

- density ρ (can change with time)
- "energy" (really–curvature) constant $\kappa = const$

A Matter-Only Universe

Simplest situation (simplest universe!): matter only

- $\rho = \rho_{\mathrm{matter}}$ only (no funny business!)
  then in sphere d, mass $M = 4\pi d^3 \rho/3 = const$
  so $\rho \propto a^{-3}$: as U exands, ρ decreases!
- total energy (curvature) zero: $\kappa = 0$

Evolution Solved

Assemble the pieces and turn the crank:

$(\dot{a}/a)^2 = 8\pi G/3\ \rho_0 a^{-3}$

$\dot{a}/a \propto a^{-3/2}$

$a^{1/2} da \propto dt$

integrate: $\int_0^a a^{1/2} da \propto \int_0^t dt$

$a^{3/2} \propto t$

$a \propto t^{2/3}$

put $t = t_0$ today

want $a(t_0) = 1$

So finally arrive at solution for matter-only Universe: $a(t) = (t/t_0)^{2/3}$

Q: fate of such a universe? i.e., what happens when $t \to \infty$?

What does it mean?
For matter-only universe, $a(t) = (t/t_0)^{2/3}$

fate: a always increases with time
$\rightarrow$ universe expands forever
as $t \rightarrow \infty$, then $a \rightarrow \infty$
universe becomes very low density!

what is t_0 = age of Universe?
since $H = \dot{a}/a = 2/3t$ always!
today: $t_0 = 2/3 \; 1/H_0 \simeq 10$ billion years

Q: is this reasonable?
compare: Earth 4.6 billion yrs
oldest stars: 10–12 billion yrs