Astro 210

Lecture 7

Sept 8, 2010

Announcements

- HW2 available, due in class next time
  15 bonus points available! Problem 4 available till Oct 1!
- as usual: Instructor office hours after class today, or by appt
  TA office hours tomorrow (Thurs) 10:30-11:30am
- **register** your iClicker; link on course webpage

- Bigshot astronomer coming next week:
  Iben Distinguished Lecturer: Tony Tyson
  lead scientist on top new telescope for 2010-2020 decade!
  "Exploring the Dark Side of the Universe"
  7pm Wed Sept 15, Foellinger; more info on course page

Last Time: Two of the All-Time Greats

Galileo: Astronomer

Kepler's Laws

1. planet orbits are ellipses, with Sun at one focus
2. orbits sweep equal areas in equal times
3. $P_{\text{yr}}^2 = a_{\text{AU}}^3$

these completely and precisely characterize planet orbits

Galileo: Physicist

isolated and studied important special cases of motion

- free body *Q: what is one? what's the motion?*
- free fall *Q: what's that? what's the motion?*

Dynamics & Gravity

Galileo not only great astronomer
but also a great physicist
paved way for Newton's dynamics by study of
two special cases of motion

1. **"free body"** – *no* external influences
natural motion: coast in straight line, const speed
$\longrightarrow$ retain current state of motion
$\longrightarrow$ bodies have **inertia**

2. **"free fall"** – when only influence is *gravity*
Galileo recognized another key motion
Demo: Tower of Pisa expt
$\longrightarrow$ constant acceleration **indep of mass**!
$a = g$, $g = 9.8\,\mathrm{m/s^2}$

Galilean free fall: constant acceleration $a = g$
So speeds change linearly with time
$v = v_0 + gt$; if $v_0 = 0$, $v = gt$
Distance traveled is quadratic in time:

$$d = \int_0^t dt' \, v(t') = \int_0^t dt' \, gt' = \frac{1}{2} g t^2 \qquad (1)$$

Ex how long does it take to drop from table to floor?
$d \sim 1\text{m} \Rightarrow t^2 = 2d/g = 2 \times 1\text{m}/9.8\,\text{m/s}^2 \sim 0.2 s^2 \Rightarrow \boxed{t \sim 0.45 \text{ s}}$

Sir Isaac Newton 1642–1727 English

Newton's Laws of Motion – "T-Shirt Review"

Newton I

a free body = no net force (i.e., no acceleration)
motion: constant velocity $\longrightarrow$ same speed **and** direction

Mathematically:
displacement $\vec{r}$, velocity $\vec{v}$, acceleration $\vec{a}$ are **vectors**

- ⋆ **displacement** $\vec{r} = (x, y, z)$, distance $r = |\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}}$
- ⋆ **velocity** $\vec{v} = d\vec{r}/dt$, speed $v = |\vec{v}|$
- ⋆ **acceleration** $\vec{a} = d\vec{v}/dt$, magnitude $a = |\vec{a}|$

Note: time derivative of vector $\vec{v}(t) = [v_x(t), v_y(t), v_z(t)]$
is $d\vec{v}/dt \equiv \dot{\vec{v}} = [\dot{v}_x(t), \dot{v}_y(t), \dot{v}_z(t)]$
where "overdot" $= d/dt$

Newton I

a free body = no net force (i.e., no acceleration)
motion: constant velocity $\longrightarrow$ same speed **and** direction

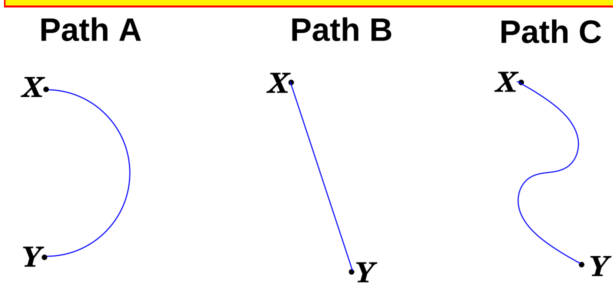
mathemathically:
acceleration $\vec{a} \equiv \dot{\vec{v}} = 0$
$\Rightarrow$ velocity $v_{\text{free}}(t) = \vec{v}_0 = const$

Newton I:
- encodes Galileo's "free body" behavior
- establishes existence of inertial frames

iClicker Poll: Acceleration

young James T. Kirk (remake version) drives from point X to Y
his motorcycle speedometer readings are unknown
maybe constant, mabye not

In which case(s) is it certain he accelerated?



A Path C only

B Paths A and C

C Paths A, B, and C

D if speed kept constant, all paths can be unaccelerated

Newton II

acceleration is proportional to force,
and inversely proportional to body's mass
$\Rightarrow a = F/m$ or $F = ma$
or $F = dp/dt$, with $p = mv$ (**momentum**)
or in 3-D:

$$\vec{p} = m\vec{v} \tag{2}$$

$$\vec{F} = d\vec{p}/dt \tag{3}$$

Newton II is machine to *predict the future!*
∞ *Q: why? how? what needed for Newtonian fortunetelling?*

Fortunetelling (and Archæology!) with Newton II:

$\dot{\vec{v}} = \vec{F}/m$: force changes speed

$\longrightarrow$ after time interval δt, velocity changed by $d\vec{v} = \vec{F}\delta t/m$

$\longrightarrow$ carries particle to new position

$\longrightarrow$ where it feels new force

$\longrightarrow$ which changes speed

...lather, rinse, repeat

So: if we know

- present position & speed (initial conditions)

then we can predict the future and reconstruct the past:

- determine the nature of the forces
- apply Newton II and turn mathematical crank
- solve particle trajectory for all time–past, present, future!

Newton III

"action-reaction"

jurisdiction: forces between objects

the rule:
when one body exerts force on another
the other body exerts force of equal magnitude
but opposite direction on the one

$$\vec{F}_{12} = -\vec{F}_{21} \tag{4}$$

$$1 \text{ on } 2 = -2 \text{ on } 1 \tag{5}$$

note magnitudes same: $|F_{12}| = F_{12} = F_{21}$

`www: jumpshot`

Newton Gravitation

Newton's **Law of Gravitation**

a force, gravity, exists between any two objects having mass

depends on masses M, m and

distance $\vec{r}$ between centers

diagram: 2-body forces

coordinates: centered at M; then force on m is

$$\vec{F} = -G\frac{Mm}{r^3}\vec{r} = -G\frac{Mm}{r^2}\hat{r} \tag{6}$$

where $r = |\vec{r}|$, and $\hat{r} = \vec{r}/r$ is a radial unit vector

G is Newton's constant: universal–applies everywhere!

but has to be determined experimentally *Q: how?*

expt: $G = 6.67 \times 10^{-11}$ m^3/kg s^2

Q: find acceleration on earth surface?

accel. on earth surface:
for body of mass m

$$
\begin{aligned}
a &= \frac{F}{m} = \frac{1}{m} G \frac{mM}{R_\oplus^2} = G \frac{M}{R_\oplus^2} = 6.67 \times 10^{-11} &(7)\\
&= 6.7 \times 10^{-11} \mathrm{m^3/kgs^2} \frac{6.0 \times 10^{24}\mathrm{kg}}{(6.4 \times 10^{6}\mathrm{m})^2} = 9.8\mathrm{m/s^2} &(8)
\end{aligned}
$$

Note:
- m cancels! as Galileo found experimentally!
- that is: inertial mass = gravitational coupling

Not obvious! no reason why need to be identical

- mass m and weight F different things

iClicker Poll: Weightlesses in Space?

Consider an astronaut orbiting Earth on the Space Shuttle
Is she weightless?

A yes

B no

C depends on whether the rockets are firing

Note larger issue:
cosmic context requires rethinking “homegrown” intuition

Gravity and Angular Momentum

For point mass, **angular momentum** defined as:
$\vec{L} = m\vec{r} \times \vec{v}$ *diagram: perspective:* $\vec{r}, \vec{p}, \vec{L}$

$$\frac{d}{dt}\vec{L} = m\dot{r} \times \vec{v} + m\vec{r} \times \dot{v} \tag{9}$$

$$= m\vec{v} \times \vec{v} + m\vec{r} \times \vec{a} \tag{10}$$

$$= \vec{r} \times \vec{F} = \vec{\tau} \quad \text{torque} \tag{11}$$

angular counterpart of Newton II:

- net (linear) force changes linear momentum
- net twisting force = torque changes angular momentum

when force is due to **gravity**, torque:

$$\frac{d}{dt}\vec{L} = \vec{\tau} = -G\frac{mM}{r^3}\vec{r} \times \vec{r} = 0 \tag{12}$$

so if force is gravity, angular momentum is **conserved!**
Q: what about gravity force gauranteed that zero?

What Keeps the Earth in Orbit?

circular orbit $\longrightarrow$ centripetal accel.
angular speed $d\theta/dt = \omega = 2\pi/P = const$, $r = const$

$$a = -\omega^2 r = -v^2/r$$

diagram: show $\vec{v}$, $\vec{r}$, $\vec{a}$

Newton II: acceleration demands net force
but Newton gravity supplies a force!
$\longrightarrow$ Newtonian gravity is crucial and necessary ingredient
for understanding the dynamics of planetary motion
but have to see how the detailed predictions
compare with observation

Program:

- assume Newtonian gravity controls planetary motion
- that is, for any planet let $\vec{F}_{\text{net}} = \vec{F}_{\text{Sun-planet}}$
- input this into Newton's laws
- turn mathematical cranks $\rightarrow$ predict orbits
- compare predictions with observation

Solutions: Orbits

For attractive inv. sqare force, oribts are cross sections of cone:
circle, ellipse, parabola, hyperbola, line
transp: oribts

Circle eccentricity $e = 0$
at each point:
$F = ma = mv_{\rm c}^2/r$
$\Rightarrow GMm/r^2 = mv_{\rm c}^2/r$
$\Rightarrow$ circular orbits have speed $\boxed{v_{\rm c} = \sqrt{\frac{GM}{r}}}$

ex: circular speed 1 AU from Sun
$v_{\rm c} = 3 \times 10^4$ m/s

Kepler from Newton

Kepler I: Orbits are ellipses

Newton: bound orbits due to gravity are ellipses: check!

Kepler II: Equal areas in equal times

Newton: consider small time interval dt

move angle $d\theta = \omega dt$

sweep area

diagram: top view: path, $d\theta, \vec{r}, \vec{v}, \vec{v}_t$

$$dA = \frac{1}{2} r^2 d\theta = \frac{1}{2} r^2 \omega dt \tag{13}$$

but $\omega = v_\theta / r$, where $\vec{v_\theta} \perp \vec{r}$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{14}$$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{15}$$

finally, $r v_\theta = |\vec{r} \times \vec{v}| = |\vec{L}|/m$
Q: why?, so

$$dA = \frac{1}{2} \frac{L}{m} dt \tag{16}$$

Woo hoo! were' home free! *Q: why?*

But $L = const$ for radial force ($\vec{r} \times \vec{F} = 0$)
so

$$\frac{dA}{dt} = \frac{L}{2m} = const \tag{17}$$

Kepler II! $\rightarrow$ comes from ang. mom. cons.!

Kepler III: $a^3 = kP^2$

Newton: can prove generally for elliptical orbits

bad news: price is lotsa algebra

good news: simple to do for circular orbits

circular $\rightarrow$ $r = a$, and $v^2 = GM/a$

but also $v = 2\pi a/P$ *Q: why?*

$$v^2 = \left(\frac{2\pi a}{P}\right)^2 = \frac{4\pi^2 a^2}{P^2} \tag{18}$$

$$= \frac{GM}{a} \tag{19}$$

$$\Rightarrow a^3 = \left(\frac{GM}{4\pi^2}\right) P^2 \tag{20}$$

check!

bounus: $k = GM/4\pi^2$ depends on mass of central object

$\rightarrow$ same k for all planets