Astro 210
Lecture 8
Sept 10, 2010

Announcements

- HW2 due now
- HW3 available, due next Friday
- HW1 graded, scores on Compass, papers back next time
- last chance! **register** your iClicker; link on course webpage

- Bigshot astronomer coming next week:
  Iben Distinguished Lecturer: Tony Tyson
  lead scientist on top new telescope for 2010-2020 decade!
  "Exploring the Dark Side of the Universe"
  7pm Wed Sept 15, Foellinger; more info on course page

Last Time: Sir Isaac Weighs In

Newton's Laws of motion

I. inertia *Q: just a special case of Newton II?*

II. $\vec{F} = m\vec{a}$ *Q: fortunetelling & archæology?*

III. action-reaction

Q: when/where/to what do Newton's laws of motion apply?

Newtonian Gravitation

Q: magnitude of gravity force between masses m and M with distance r?

Q: direction of the force?

Newton's laws sweeping in scope!
as stated they, should apply to
any motion of *any* object of *any* size, shape, or speed, with *any* forces acting at *any* time

in fact: for all of everday life,
and indeed for much of the cosmos and its history,
Newton's laws work fantastically well!

but we will see soon: they are not the last word
under very extreme (and very interesting!) conditions
Newton's laws fail...

but to understand the significance of Newton's limitations
first need to appreciate his most spectacular success...

Angular Momentum

For point mass, **angular momentum** defined as

$$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} \tag{1}$$

where $\times$ is vector cross product *diagram: perspective:* $\vec{r}, \vec{p}, \vec{L}$

What is time change of angular momentum?

$$\begin{aligned} \frac{d}{dt}\vec{L} &= m\dot{r} \times \vec{v} + m\vec{r} \times \dot{v} && (2) \\ &= m\vec{v} \times \vec{v} + m\vec{r} \times \vec{a} && (3) \\ &= \vec{r} \times \vec{F} = \vec{\tau} \quad \text{torque} && (4) \end{aligned}$$

angular counterpart of Newton II

- $d\vec{p}/dt = \vec{F}$: net (linear) force changes linear momentum
- $d\vec{L}/dt = \vec{\tau}$: net twisting force = torque changes angular momentum

iClicker Poll: Orbits and Angular Momentum I

For an object in a Keplerian (elliptical) orbit around the Sun
Consider the **directions** of angular momentum at *perihelion*, and *aphelion*

How do these compare?

A the two vectors are parallel

B the two vectors are antiparallel

C none of the above

Gravity and Angular Momentum

in general:
angular momentum $\vec{L} = m\vec{r} \times \vec{v}$
torque $\vec{\tau} = \vec{r} \times \vec{F} = d\vec{L}/dt$

when force is due to **gravity**, torque is:

$$\frac{d}{dt}\vec{L} = \vec{\tau} = -\vec{r} \times G\frac{mM}{r^3}\vec{r} = 0 \qquad (5)$$

so if force is gravity:

- torque is *zero*! ...and so
- angular momentum is **conserved!**

Q: what about gravity force gauranteed that zero?

What Keeps the Earth in Orbit?

circular orbit $\longrightarrow$ centripetal accel.

angular speed $d\theta/dt = \omega = 2\pi/P = const$, $r = const$

$$a = -\omega^2 r = -v^2/r$$

diagram: show $\vec{v}$, $\vec{r}$, $\vec{a}$

Newton II: acceleration demands net force

but Newton gravity supplies a force!

$\longrightarrow$ Newtonian gravity is crucial and necessary ingredient
for understanding the dynamics of planetary motion
but have to see how the detailed predictions
compare with observation

Program:

- assume Newtonian gravity controls planetary motion
- that is, for any planet let $\vec{F}_{\text{net}} = \vec{F}_{\text{Sun-planet}}$
- input this into Newton's laws
- turn mathematical cranks $\rightarrow$ predict orbits
- compare predictions with observation

Solutions: Orbits

For attractive inv. sqare force, oribts are cross sections of cone:
circle, ellipse, parabola, hyperbola, line
transp: oribts

Circle eccentricity $e = 0$
at each point:
$F = ma = mv_{\rm c}^2/r$
$\Rightarrow GMm/r^2 = mv_{\rm c}^2/r$
$\Rightarrow$ circular orbits have speed $\boxed{v_{\rm c} = \sqrt{\frac{GM}{r}}}$

ex: circular speed 1 AU from Sun
$v_{\rm c} = 3 \times 10^4$ m/s

Kepler from Newton

Kepler I: Orbits are ellipses

Newton: bound orbits due to gravity are ellipses: check!

Kepler II: Equal areas in equal times

Newton: consider small time interval dt

move angle $d\theta = \omega dt$

sweep area

diagram: top view: path, $d\theta, \vec{r}, \vec{v}, \vec{v}_t$

$$dA = \frac{1}{2} r^2 d\theta = \frac{1}{2} r^2 \omega dt \tag{6}$$

but $\omega = v_\theta / r$, where $\vec{v_\theta} \perp \vec{r}$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{7}$$

$\Rightarrow$ swept area

$$dA = \frac{1}{2} r^2 \frac{v_\theta}{r} dt = \frac{1}{2} r v_\theta dt \tag{8}$$

finally, $r v_\theta = |\vec{r} \times \vec{v}| = |\vec{L}|/m$
Q: why?, so

$$dA = \frac{1}{2} \frac{L}{m} dt \tag{9}$$

Woo hoo! were' home free! *Q: why?*

But $L = const$ for radial force ($\vec{r} \times \vec{F} = 0$)
so

$$\frac{dA}{dt} = \frac{L}{2m} = const \tag{10}$$

Kepler II! $\rightarrow$ comes from ang. mom. cons.!

iClicker Poll: Orbits and Angular Momentum II

Consider two objects at 1 AU:

- a planet in a circular orbit
- a spacecraft moving directly towards the Sun

Which has the larger *magnitude* of angular momentum?

A the planet

B the spacecraft

C either, depending on the speed of the spacecraft

Kepler III: $a^3 = kP^2$

Newton: can prove generally for elliptical orbits

bad news: price is lotsa algebra

good news: simple to do for circular orbits

circular $\rightarrow r = a$, and $v^2 = GM/a$

but also $v = 2\pi a/P$ *Q: why?*

$$v^2 = \left(\frac{2\pi a}{P}\right)^2 = \frac{4\pi^2 a^2}{P^2} \quad (11)$$

$$= \frac{GM}{a} \quad (12)$$

$$\Rightarrow a^3 = \left(\frac{GM}{4\pi^2}\right) P^2 \quad (13)$$

check!

bounus: $k = GM/4\pi^2$ depends on mass of central object

$\rightarrow$ same k for all planets