Astro 210

Lecture 9

Sept 13, 2010

Announcements

- HW3 due in class Friday
  hardcopies available–note they are **twosided**
- HW2 Q4 (10 bonus points) available till Oct 1
- HW1 back today; scores on Compass
- Participation (iClicker) scores posted on Compass

- Bigshot astronomer in town this week
  Iben Distinguished Lecturer: Tony Tyson
  lead scientist on top new telescope for 2010-2020 decade!
  "Exploring the Dark Side of the Universe"
  7pm Wed Sept 15, Foellinger; more info on course page

Last time: Kepler from Newton

solve $\vec{F} = m\vec{a} = m\ddot{\vec{r}}$ with $\vec{F} = -GMm/r^2\ \hat{r}$

gives back Kepler's laws, and so

- agrees precisely with observed planet orbits
- also explains how orbits arise from gravity
- and gives, e.g., circular speed: $\boxed{v_{\text{c}} = \sqrt{\frac{GM}{r}}}$
- and updates Kepler III: $a^3 = \left(\frac{GM}{4\pi^2}\right)P^2$

Energy

For "test" particle m moving due to gravity of M
Gravitational potential energy: *Q: why "potential"?*
$PE = -GMm/r$

Kinetic energy:

$$KE = \frac{1}{2}m\vec{v}^2 = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \frac{1}{2}m\dot{\vec{r}}^2 \tag{1}$$

Total energy:
$TE = KE + PE = -GMm/r + \frac{1}{2}m\dot{\vec{r}}^2$
key result: $d(TE)/dt = 0$
$\longrightarrow$ total energy conserved!
that is: value of TE the same for all time!

Orbits Revisited

Bound orbits (circle & ellipse): in polar coordinates

$$r(\theta) = \frac{(1-e^2)a}{1+e\cos\theta} \tag{2}$$

Circle radius $r = a = const$, eccentricity $e = 0$

recall: circular orbit has constant speed $v_c^2 = GM/r$

$$\begin{aligned} PE &= -\frac{GMm}{r} \quad < 0 && (3) \\ KE &= \frac{1}{2}mv_c^2 = \frac{1}{2}m\frac{GM}{r} = \frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}PE && (4) \\ \Rightarrow TE &= KE + PE = PE/2 = -|PE|/2 < 0 && (5) \end{aligned}$$

$TE < 0$: negative? yes!

Q: what does it mean to have negative energy?

for orbiting system $TE < 0$:
$\rightarrow$ have to *supply* energy to system to break it apart

Why? when particles are *at rest* and "very" *far apart*
$KE = mv^2/2 = 0$
$PE = GMm/r \rightarrow 0$ *Q: how far apart is this?*
and so $TE = KE + PE = 0$: zero total energy
But if start in closed orbits (circular or elliptical): $TE < 0$
$\rightarrow$ To "break" the system from closed orbits, must *add* energy
But energy is conserved $\rightarrow$ not spontaneously added
so system is **bound**
$\Rightarrow$ can't fall apart without external influence

Note: $KE = -PE/2 = |PE|/2$ generally true for
gravitating systems in equilibrium:
"virial theorem"

ellipse: semimajor axis a, eccentricity $0 < e < 1$

turns out: TE depends only on a, not e

from cons of energy

$TE = -GMm/r + \frac{1}{2}mv^2 = -GMm/2a < 0 \rightarrow$ bound

can show

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{6}$$

"vis viva" equation ("life force")

discovered prior to concept of energy

handy: gives total speed v at any radius r

Q: at which r is $v = 0$? how does this work for a circular orbit?

Q: for a given orbit (fixed e), when is v max?

Unbound Orbits

Note that both parabolic and hyperbolic orbits are not periodic – do not close on themselves "one-way ticket" past the central object

Parabola

$e = 1$

$$r = \frac{2p}{1 + \cos\theta} \tag{7}$$

p is distance of closest approach

for parabolic orbit:

$TE = 0$ exactly! $\rightarrow KE = -PE$ exactly! very special case!

$\Rightarrow GM/r = \frac{1}{2}v^2$

So at $r = \infty$, $v = 0$

to have this orbit, launch from r with speed

$v_{\text{launch}} = \sqrt{2GM/r}$

iClicker Poll: Orbits

given: test particle m, at distance r from gravitating body M
for test particle to have total energy $TE = 0$
launch from r with speed $v_0 = \sqrt{2GM/r}$

Q: what happens if launch with speed $v > v_0$?

A particle will be in a bound orbit: circle or ellipse

B particle will be unbound, with speed $v \to 0$ as $r \to \infty$

C particle will be unbound, with speed $v > 0$ as $r \to \infty$

Q: why is v_0 a special speed?

Escape Speed

At radius r, define **escape speed** $v_{\text{esc}} = \sqrt{2GM/r}$

- if launch from r with $v_{\text{launch}} < v_{\text{esc}}$
  then $TE < 0$: fall back! (elliptical orbit)
- if launch from r with $v_{\text{launch}} > v_{\text{esc}}$
  then $TE > 0$: escape "easily": $v > 0$ at $r = \infty$
- if launch from r with $v_{\text{launch}} = v_{\text{esc}}$ exactly
  thyen $TE = 0$ exactly, "just barely" escape

So: escape speed is *minimum speed* needed to leave
a gravitating source

Example: escape speed from earth
$v_{\text{esc}} = 11$ km/s = 25,000 mph!
predict the future: if toss object with $v < 25,000$ mph, falls back
but if $v > 25,000$ mph *Q: example?* never returns!

finally, the more "generic" unbound orbit:

hyperbola

$$r(\theta) = \frac{(e^2 - 1)a}{1 + e\cos\theta} \tag{8}$$

$e > 1$, $TE > 0$

$v > 0$ at $r = \infty$: nonzero speed far from M

Recall: at large r, hyperbola $\rightarrow$ *straight line*

But Newton says: $d\vec{v}/dt = -GM/r^2\ \hat{r}$

so as $r \rightarrow \infty$, then $d\vec{v}/dt \rightarrow 0$

$\Rightarrow$ gravity negligible, $\vec{v} \rightarrow$ const: *free body*=straight line!

orbit of unbound "flyby":

distant nearly free body $\rightarrow$ passing: pulled toward M

$\rightarrow$ distant deflected nearly free body

Testing Newton's Gravity

Moons of Juptier: obey Kepler's laws
$\longrightarrow$ Jupiter's gravity works like Sun's, Earth's

1830's: Uranus observed orbit did not follow predictions of Newtonian solar system model
$\Rightarrow$ the death Newton's gravity?

recall: theory must explian *all* data, not just some!
so despite Newton's great job with planets, moons
even *one clear failure is enough*

Q: so do we have to throw out Newtonian gravity?
Q: why hesitant to throw out?
Q: if not abandon, what's another solution to the problem?

iClicker Poll: Uranus Discrepancy

1830's Problem: *measured* Uranus orbit doesn't match preditions of Newtonian Gravity *theory*

Vote your conscience!

Which seems more likely to you?

A Newton's gravity theory *correct*, but not all gravity sources had been included

B Newton's gravity theory *incorrect* (or at least incomplete)

Q: what experiment/observation would tell which is right?

maybe haven't included all sources of gravity?
maybe unknown object causes U's deviations?
$\Rightarrow$ a new planet?

if unknown object, could predict where should be
did this, looked. saw:
`www: Neptune`
1846: Neptune found at right position
$\triangleright$ *predicted* by Newton's gravity

very impressive! victory snatched from jaws of defeat!
triumph of Newtonian dynamics and gravity

many other confirming observations
`www: binary star orbits`