

NAME: _____

Astronomy 350

Fall 2011

HOUR EXAM 2
October 28, 2011

1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
2. Write your name above.
3. Show all of your work, and indicate clearly your final answer! A correct final answer may not receive credit if no work is shown.
4. Budget your time! Don't get stalled on any one question.
5. Short answer questions can be answered in 1-2 sentences, unless indicated otherwise. If you are writing paragraphs, you may have misread or misunderstood the question.
6. For your reference there are constants listed below.
7. The total number of points on the exam is 100.

Possibly Useful Information

Note that a symbol may take different meanings in different equations.

$$\Delta x = v \times \Delta t$$

$$\Delta v = a \times \Delta t$$

$$P_{\text{yr}}^2 = a_{\text{AU}}^3$$

$$F = ma$$

$$F = Gm_1m_2/R^2$$

$$KE = \frac{1}{2}mv^2$$

$$PE = -Gm_1m_2/R$$

$$v_{\text{esc}} = \sqrt{2GM/R}$$

$$M = v_{\text{circ}}^2 R/G$$

$$F = L/4\pi R^2$$

$$d = 1 \text{ pc}/p_{\text{arcsec}}$$

$$L \propto M^4$$

$$\tau = 10^{10} \text{ yr } (M/M_{\odot})^{-3}$$

$$N = N_{\text{init}} 2^{-t/T}$$

$$\Delta t_{\text{obs}} = \Delta t_{\text{rest}} / \sqrt{1 - v^2/c^2}$$

$$L_{\text{obs}} = L_{\text{rest}} \sqrt{1 - v^2/c^2}$$

$$E = mc^2 / \sqrt{1 - v^2/c^2}$$

$$KE = E - mc^2$$

$$R_{\text{Sch}} = 2GM/c^2$$

$$R_{\text{Sch},\odot} = 2GM_{\odot}/c^2 = 3 \text{ km}$$

$$\Delta t_{\text{obs}}/\Delta t_{\text{em}} = \lambda_{\text{obs}}/\lambda_{\text{em}} = \sqrt{\frac{1 - R_{\text{Sch}}/r_{\text{obs}}}{1 - R_{\text{Sch}}/r_{\text{em}}}}$$

$$z = (\lambda_{\text{obs}} - \lambda_{\text{em}})/\lambda_{\text{em}}$$

$$v = cz$$

$$v = H_0 r$$

$$G = 6.7 \times 10^{11} \text{ m}^3/\text{kg s}^2$$

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

$$1 \text{ pc} = 3.1 \times 10^{16} \text{ m} = 3.3 \text{ lyr}$$

$$1 \text{ kpc} = 10^3 \text{ pc} = c \times (3300 \text{ yr})$$

$$M_{\odot} = 2.0 \times 10^{30} \text{ kg}$$

$$M_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg}$$

$$L_{\odot} = 3.8 \times 10^{26} \text{ Watts}$$

$$\tau_{\odot} = 10^{10} \text{ yr} = 10 \text{ billion yrs}$$

$$H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$t_{\text{H}} = 1/H_0 = 14 \text{ billion years}$$

$$d_{\text{H}} = c/H_0 = 4200 \text{ Mpc}$$

Multiple Choice

Circle the single *best* answer for each question. Be sure clear indicate only **one** final answer.

1 . **[5 points]** Which of the following must be *forbidden* by the Cosmological Principle?

- (a) a homogeneous universe which is rotating about an axis
- (b) a universe in which all matter is contained inside a sphere of radius 4000 Mpc
- (c) a universe containing both spiral and elliptical galaxies
- (d) **both (a) and (b)**
- (e) all of (a), (b), and (c)

(a) is not isotropic—there is a special direction. (b) is not homogeneous—there is an edge to the universe. (c) is fine as long as the mix of spirals and ellipticals is the same everywhere, as it is in our universe.

2 . **[5 points]** At Fermilab, an unstable particle is created, and moves at speed $v = 0.999c$ relative to the laboratory. In the lab, the particle is observed to decay after $\tau_{\text{obs}} = 10^{-6}$ sec. An observer *at rest* relative to the particle would measure the decay time to be

- (a) 10^{-6} sec
- (b) $< 10^{-6}$ sec
- (c) $> 10^{-6}$ sec
- (d) an observer at rest relative to the particle would never see it decay

Moving clocks appear to run slowly—this is time dilation. So a moving muon will seem to take longer to decay than one at rest. Indeed, this is the example given in class as evidence for time dilation.

3 . **[5 points]** A galaxy is observed to recede with speed $v = 7200$ km/s. According to Hubble's law, the galaxy lies at a distance

- (a) 0.01 Mpc
- (b) **100 Mpc**
- (c) 72 Mpc
- (d) 720 Mpc
- (e) 4200 Mpc

Hubble's law says $v = H_0 r$, so $r = v/H_0$.

4 . **[5 points]** Two galaxies move relative to us according to Hubble's law. If the galaxies show *different* redshifts, then

- (a) they *must* lie at the same distance from us
- (b) **they cannot lie at the same distance from us**
- (c) they *may* lie at the same distance from us

Hubble says $v = H_0 r$, and $v = cz$, so different z means different v which means different r .

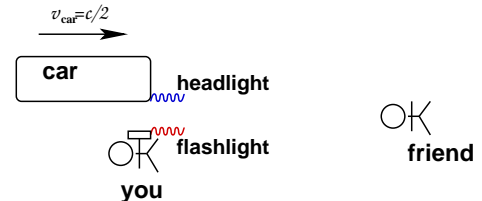
5 . **[5 points]** Consider a set of observers, “the Simpsons,” all described by Special Relativity (i.e., you may ignore effects of gravity); one of them happens to be named Bart. These observers move at different speeds relative to each other. Which statement(s) will *all* observers (including Bart) *agree* on?

- (a) Bart is at rest
- (b) Bart's speed relative to me is less than c

- (c) Bart's watch never appears to tick faster than my identical watch
- (d) **both (b) and (c)**
- (e) all of (a), (b), and (c)

Moving clocks appear to run slowly, so a moving clock will never appear to tick faster than one at rest.

6 . [5 points] See diagram at right. As you stand at rest outside the Astronomy building, a sports car zooms by you, moving at speed $v_{\text{car}} = c/2$. Just as the car passes you, you turn on a flashlight and the car turns on its headlights (i.e., both light pulses are emitted at the same place and time). You friend, at rest outside the Physics building, will see



- (a) the car's headlights first
- (b) your flashlight first
- (c) **both lights at the same time**

The speed of light is the same for all observers.

- 7 . [5 points] Which of these is evidence for a supermassive black hole at the center of our Galaxy?
- (a) the innermost 1 kpc of the Galaxy is empty because the black hole has consumed all nearby stars
 - (b) the number of stars at the Galactic center is seen to constantly decrease as they fall into the black hole
 - (c) **tracking the orbits of stars at the Galactic center implies a large unseen mass concentration**

8 . [5 points] In the distant Bieber galaxy, hydrogen atoms emit photons with two wavelengths. As measured by observers in galaxy Bieber, these photon wavelengths are in the ratio $\lambda_1/\lambda_2 = 4$ exactly. When these photons are observed on earth, the ratio λ_1/λ_2 will be:

- (a) **exactly 4**
- (b) greater than 4
- (c) less than 4
- (d) either (b) or (c), depending on the distance to Galaxy Bieber

If a galaxy is at redshift z , then a photon emitted at wavelength λ_{em} is observed at wavelength $\lambda_{\text{obs}} = (1+z)\lambda_{\text{em}}$. Thus means that $\lambda_{1,\text{obs}} = (1+z)\lambda_{1,\text{em}}$. and $\lambda_{2,\text{obs}} = (1+z)\lambda_{2,\text{em}}$. And so we find that

$$\frac{\lambda_{1,\text{obs}}}{\lambda_{2,\text{obs}}} = \frac{(1+z)\lambda_{1,\text{em}}}{(1+z)\lambda_{2,\text{em}}} = \frac{\lambda_{1,\text{em}}}{\lambda_{2,\text{em}}} \quad (1)$$

and so the wavelength *ratio* does not change even though each wavelength individually gets longer!

Short Answer

Answer briefly but completely. Your responses should not require more than 1-2 sentences.

1. Black holes, dark matter, and gravitational lensing.

- (a) [5 points] Give two properties of black holes that make them good dark matter candidates. They are dark: black holes emit no light. And they are matter: black holes have mass and gravitate.

- (b) **[10 points]** Briefly explain how can we use gravitational lensing to detect black holes in our Galaxy's halo.

We can monitoring many stars in a nearby galaxy. Then when a black hole in our halo passes between us and a star in the nearby galaxy, the star's light will be gravitationally lensed and magnified; we can detect this magnification as gravitational microlensing.

- (c) **[5 points]** Searches as described in part (b) show that our Galactic halo is not made of black holes. Briefly (1-2 sentences) explain why this result makes it unlikely that black holes are the dark matter in the halos of other Galaxies.

The universe is homogeneous, so the nature of the dark matter in our galaxy should be the same as the dark matter in other galaxies.

2. The cosmological principle.

- (a) **[5 points]** Briefly (1-2 sentences) summarize the cosmological principle.

(1) The universe is homogeneous—at any given moment, the density and composition of the universe are the same in all points in space.

(2) The universe is isotropic—it looks the same in all directions.

- (b) **[10 points]** In class, it was demonstrated that, in an expanding universe, all observers find that galaxies move according to Hubble's Law. Briefly explain why the cosmological principle demands that this must be the case.

Homogeneity and isotropy mean that the universe must look the same to all observers. Thus if one observer measures Hubble's law, all observers must measure the same thing.

3. General Relativity: Black Holes. Two astronauts, Ashton and Demi, each hover near a black hole, but possibly at different distances. Ashton notices that in the time it take his watch to tick off 1 second, Demi's watch appears to tick off just 0.1 seconds.

- (a) **[5 points]** Who is closer to the black hole? Briefly explain your reasoning.

Time “elapses slower near a black hole” in that observers nearer to a black hole agree that observers farther from a black hole have clocks that run faster. Of course, all observers believe their own clocks run at normal speed, and it is the other observers whose clocks are behaving strangely.

- (b) **[5 points]** From Demi's point of view, when her watch ticks off 1 second how many seconds does Ashton's watch tick off? Briefly justify your answer.

We are given that $\Delta t_{\text{demi}} = 0.1\Delta t_{\text{ashton}} = \Delta t_{\text{ashton}}/10$. But this also means that $\Delta t_{\text{ashton}} = 10\Delta t_{\text{demi}}$, so if Demi sees her clock tick off $\Delta t_{\text{demi}} = 1$ sec, then she will see Ashton's clock tick off $\Delta t_{\text{ashton}} = 10 \times 1 \text{ sec} = 10 \text{ sec}$.

4. Special Relativity.

- (a) **[5 points]** Briefly explain why a massive (i.e., $m \neq 0$) particle can't be accelerated to speed c .

The total energy of a massive particle is related to its mass and speed by $E = mc^2/\sqrt{1 - v^2/c^2}$. This tells us that as $v \rightarrow c$, then $E \rightarrow \infty$: to accelerate to the speed of light requires infinite energy! This is not possible, so massive particles must always move with speeds less than c . Thus c is the maximum speed in nature, and is only attained by light or any other massive particle.

- (b) A recent measurement has reported particles (neutrinos) moving at speed $v = 1.0000245c$.

- i. **[5 points]** *If special relativity is correct, what would that mean for this result?*
Since special relativity says that c is the speed limit, if SR is true then this result is incorrect—there has been an error in the experiment.
- ii. **[5 points]** *If this result is correct, what would that mean for special relativity?*
If the experiment is true, then it is possible to have $v > c$. This contradicts special relativity, so special relativity must be incomplete/wrong.