Astronomy 350 Fall 2011
Homework #4
Due in class: Friday, Sept. 30

1. *Properties of Stars.*

   (a) [**5 points**]. What is the best evidence today that the Sun is undergoing nuclear reactions? Explain why the evidence is very strong.

   (b) [**5 points**]. The evidence in part (b) is not available for other stars. How do we know (and we *do* know!) that these also are undergoing nuclear reactions?

   (c) [**5 bonus points**]. if the universe starts with H and He and no heavier elements ("metals"), describe how the amounts of H, He, and metals each change with time as generations of stars are born and die

2. [**5 points**]. High in the evening summer sky, three stars are quite bright: Deneb, Altair, and Vega. These together form a triangle, known as the Summer triangle. Do **one** of the following problems.

   (a) The ratio of fluxes for Vega and Deneb is $F_{\rm Vega}/F_{\rm Deneb} = 3.1$. These stars have parallaxes $p_{\rm Vega} = 0.13$ arc sec, $p_{\rm Deneb} = 0.0023$ arc sec. Show which has a higher luminosity.

   (b) *For the Technorati.* Stellar brightnesses are measured and tabulated in the "magnitude" scale. A magnitude is a logarithmic measure of flux. defined so that a magnitude *difference* is *ratio* of fluxes:

   $$m_2 - m_1 = -2.5 \log_{10} \frac{F_2}{F_1} \tag{1}$$

   with unfortunate prefactor of 2.5, and even more unfortunate $-$ sign, so large magnitudes correspond to faint objects. In the Summer triangle, the stars have magnitudes $m_{\rm Altair} = 0.77$, $m_{\rm Deneb} = 1.26$, and $m_{\rm Vega} = 0.04$. Their parallaxes are $p_{\rm Altair} = 0.20$ arc sec, $p_{\rm Deneb} = 0.0023$ arc sec, and $p_{\rm Vega} = 0.13$ arc sec. Use these data to place the stars in order of luminosity, and find the ratio $L_{\rm Altair} : L_{\rm Deneb} : L_{\rm Vega}$.

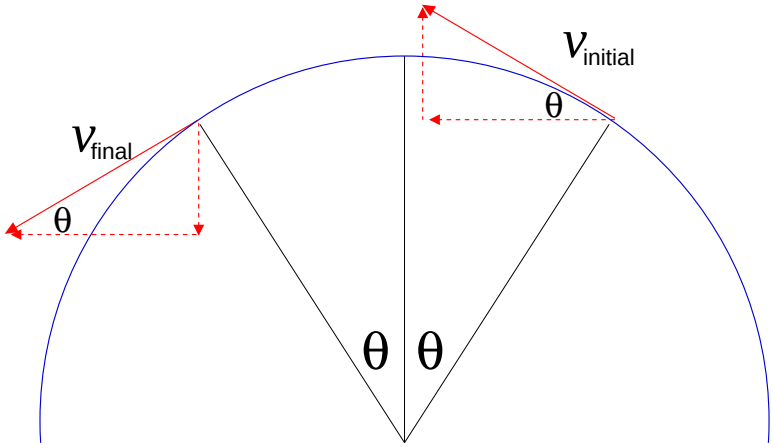
3. *Circular Motion, Acceleration, and Dark Matter.* In class it was asserted that an object in circular orbit around a spherical gravitating body has an orbit radius R and speed v that together give the gravitating mass

   $$M = \frac{v^2 R}{G} \tag{2}$$

   Here we will show how this comes about.

   (a) [**5 points**]. An important aspect of the problem is the nature of circular motion. In particular, circular motion involved constant acceleration, directed towards the center, with magnitude $a_{\rm circ} = v^2/R$. Do **one** of the following.

Option 1: In the diagram at right, we see the velocity at two instants. Show that the *change* in velocity in the x (horizontal) direction is zero, while there is a nonzero change in the velocity in the y (vertical) direction, and that the change is directed towards the center. Note: for circular motion at constant speed, it follows that the magnitudes $v_{\text{initial}} = v_{\text{final}}$ even though the directions are different.



Option 2 (for the technorati): Show that for circular motion, $\vec{a} = v^2/r\ \hat{r}$. One approach is use the kinetic energy in polar coordinates, $T = \frac{1}{2}mv^2 = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2)$ along with the Euler-Lagrange equations. Another is to use the Cartesian vector equation $\vec{r}(t) = [x(t), y(t)] = [r\cos(\omega t), r\sin(\omega t)]$ where $r = |\vec{r}| = \sqrt{x^2 + y^2} = const$, along with the definitions $\vec{v} = \dot{\vec{r}}$ and $\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$.

(b) [**5 points**]. Since the motion is due to gravity, the acceleration from equation (2) has to be due to the gravitational force. If this is the case, show that $M = v^2R/G$.

(c) [**5 points**]. The Sun is moving at a distance of about $R = 8$ kpc and a speed of about $v = 200$ km/s around the Galactic center. Using these data, calculate the mass M of material inside the orbit of the Sun. Be careful with units! Probably the best thing is to convert everything to meters, kg, and seconds first. Finally, express your answer in terms of the mass $M_\odot$ of the Sun, i.e., find $M/M_\odot$. Comment on the significance of this result.

(d) [**5 points**]. How can the result from part (c) be used to tell that our Galaxy contains dark matter? What other information is needed to come to this conclusion?

(e) [**5 points**]. What tells us dark matter is "dark"? What tells us it is "matter"?

(f) [**5 points**]. What is a possible candidates dark matter? What about it makes it a good candidate–that is, how does it fit the definition of dark matter from part (e)?

(g) [**5 points**]. It is possible dark matter does not exist. If so, what is an alternate scientific explanation of the observations that led to the idea of dark matter?