Astronomy 350 Fall 2011
Homework #5
Due in class: Friday, Oct. 7

1. *A Gut Feeling for Dark Matter*

   (a) [**5 points**]. Mass density is defined as $\rho = M/V$, the ratio of mass to volume. In HW4, you hopefully found that at the Sun's $R = 8$ kpc distance from the Galactic center, the mass enclosed in a sphere of radius R is about $M = 10^{11} M_\odot$. Using this, find the mass density of dark matter $\rho_{\rm dm}$ at the location of the Earth. Express your answer in kg/m^3. Compare your result to the density of liquid water, which is $\rho_{\rm water} = 1000$ kg/m^3.

   (b) [**5 points**]. If dark matter takes the form of elementary particles, as many cosmologist think, current theories favor an individual particle mass of $m_{\rm dm} \approx 100 m_{\rm proton} = 1.6 \times 10^{-25}$ kg. If this is the case, find the *number* of dark matter particles per cubic meter at the location of the Earth.

   (c) [**5 points**]. Now make a rough estimate the volume of $V_{\rm me}$ your body. You are welcome to make the "cylindrical human" approximation, or even the more drastic "spherical human" approximation–a rough ballpark number is all that is needed here, not any sort of ultrapersonal disclosure! Express your answer in cubic meters (m^3).

   (d) [**5 points**]. Finally, combine the results from part (b) and part (c) to estimate the number of dark matter particles in your body at any given time. Comment on your result.

   (e) [**5 points**]. We expect that typically, a dark matter particle will move relative to us with about the Sun's orbit speed of 200 km/s. Given this, about how long does any individual dark matter particle stay in your body? How do you reconcile this with the result in part (d)?

2. *Particle Dark Matter*

   (a) [**5 points**]. If dark matter is in the form of elementary particles, it has to be weakly interacting. To see this, imagine the contrary–that dark matter could readily collide with ordinary matter. Explain what Earth-based and solar system experiments could be done to rule out such particles.

   (b) [**5 bonus points**]. Dark matter not only has to be neutral, it also has to be electrically neutral. To see this, imagine that the DM is positively charged, with the same charge as a proton. How would an electron respond to such a particle? How would a dark matter + electron system reveal itself in lab experiments?

3. *Superluminal Neutrinos?* As discussed in class, the OPERA experiment has recently announced a measurement which seems to find objects–neutrinos–moving faster than the speed of light. Several technical and non-technical accounts of the experiment and its implications are linked in the webpage for Monday Sept 26.

   (a) [**5 points**]. Briefly explain the basics of the experiment. Where are the neutrinos produced? Where are they detected? How is it possible for the neutrinos to travel hundreds of miles underneath the Earth?

The "time of flight" is the time between the neutrino production and detection. How is this used to measure neutrino speed?

(b) [**5 points**]. The OPERA experiment reports neutrino speed v in terms of the quantity $(v - c)/c = 2.48 \times 10^{-5}$. Solve for neutrinos speed in terms of the speed of light, that is, write $v = Xc$ and find the value of X.

You should find that v is very nearly c. Since the two speeds are so close, is the result still important? What current ideas would have to change as a result?

(c) [**5 points**]. Supernova explosions produce huge numbers of neutrinos–in fact, *most* of the explosion energy is radiated in neutrinos, not in electromagnetic radiation.

In February of 1987, a supernova explosion went off in the Large Magellanic Cloud, a small galaxy which orbits the Milky Way. This event, supernova 1987A, was detected by ordinary (electromagnetic) telescopes, but also by neutrino detectors.

Let us assume the neutrino speed from the OPERA experiment is appropriate for the neutrinos from SN 1987A. The photons from the explosion by definition move at speed c. The supernova was at a distance $d = 50$ kpc = 50,000 pc. Write an expression (that is, an equation just using variables, not plugging in numbers yet) for the time t_γ that light took to travel this distance; also find and expression for the time t_ν that neutrinos took to travel this distance. Combine these for an expression for the *difference* in times of flight from the supernova. Finally, use your formula for the case of SN 187A, plugging in numbers. If the photons and neutrinos were emitted at the same time, how much earlier would we expect the neutrinos to arrive?

(d) [**5 points**]. In fact, neutrinos seen a few hours *before* the SN 1987A explosion. How strongly does this contradict your prediction from part 3c?

In fact, the neutrinos from SN 1987A had much lower energy than those detected by OPERA. It could be that neutrino speeds depend on their energies. If so, would the low-energy 1987A neutrinos need to be slower or faster than the high-energy OPERA neutrinos?