Astronomy 350 Fall 2011
Homework #9

Due in class: Friday, Nov. 11

1. *Dark Energy.*

   (a) [**5 points**]. Why is dark dark energy "dark"? Why it is "energy?"

   (b) [**5 points**]. What is the difference between the cosmological constant and dark energy?

   (c) [**5 points**]. How can we test whether our universe has a cosmological constant?

   (d) [**5 points**]. What would it mean if a cosmological constant *can* explain the acceleration of the universe? What question(s) would that raise?

   (e) [**5 points**]. What would it mean if a cosmological constant *cannot* explain the acceleration of the universe? What question(s) would that raise?

2. *The Onset of Acceleration.* Observations of the cosmic expansion rate show that cosmic acceleration began relatively recently, while at early times the universe was decelerating. Models for cosmic acceleration must explain this; we will see how this works in a universe that contains matter and a cosmological constant Λ.

   (a) [**5 points**]. In a universe with matter density $\rho_{\rm matter}$ and a cosmological constant, the Friedmann acceleration equation is

   $$\text{acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi}{3} G \rho_{\rm matter} + \frac{1}{3}\Lambda \qquad (1)$$

   Explain the difference between: (a) the acceleration of a universe with $\rho_{\rm matter} = 0$ but $\Lambda > 0$, and (b) a universe with $\rho_{\rm matter} > 0$ but $\Lambda = 0$. Then explain what we learn about Λ from the fact that today the universe is known to be accelerating.

   (b) [**5 bonus points**]. Show that eq. (1) can be written as

   $$\text{acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi}{3} G \left(\frac{\rho_{\rm matter,0}}{a^3} - 2\rho_\Lambda \right) \qquad (2)$$

   where $\rho_{\rm matter,0}$ is the matter density today (when $a = 1$), and $\rho_\Lambda = \Lambda/8\pi G$.

   (c) [**5 points**]. Now consider early times, when the universe was "smaller" and so $a < 1$. Consider eq. (2), and explain why the universe should have been decelerating at early times. What changed since then to make the universe accelerate now?

   (d) [**5 points**]. Using eq. (2), solve for the value of a when the universe made a transition from deceleration to acceleration; this occurs when acceleration = 0. Then use the observed value $\rho_\Lambda/\rho_{\rm matter,0} \approx 7/3$ to find the value of the scale factor at this transition. Also find the redshift $z = \frac{1}{a} - 1$ of this transition. We routinely observe galaxies at redshift $z > 3$ today–was the universe accelerating or decelerating then?

3. [**5 points**]. *Cosmic Inventory.* What are the three largest contributors to the density of the universe today? What fraction of the cosmos today is made of material that has been studied in a laboratory? Briefly comment on this result.

4. [**5 points**]. *The CMB*. What is the CMB? In what ways is it cosmic? microwave? background? radiation?