Astro 350

Lecture 19

Oct. 7, 2011

Announcements:

- HW5 due
- HW6 out, due next Friday
- Discussion Question 6 posted, due next Wednesday

Last time: Special Relativity

- train + lightning experiment: lessons?
- moving clock experiment: lessons?

Train: disagreement about whether events are simultaneous
$\rightarrow$ "at the same time" is not a universal concept
$\rightarrow$ "relativity of simultaneity"
$\rightarrow$ "universal time" doesn't exist, depends on motion

Moving clock: disagreement about tick duration
- moving clock appears to run slowly (tick lasts longer)
- "time dilation"
- time does not "flow" at universal rate, depends on motion

Q: who's right in all these disagreements?

Everybody's right! that is,

- all observations correctly reported
- real problem is deeper:
  Aristotelian notion of universal space, time are *invalid*

⋆ Space, time depend on state of motion of observers
⋆ but *no* observer *ever* sees her/himself as the wierdo
  i.e., your (nearby, at rest relative to you) clocks & yardsticks
  never appear weird to you
⋆ bizarreness only can arise when looking at things moving fast
  and/or at a distance

More later on what we *can* all agree on...

Time Dilation in the Real World

high energy particles from space ("cosmic rays")
collide with Earth's atmosphere
produce unstable particles: muons μ seen at ground
in lab, at rest: muons decay after $t_{\text{decay}} = 2 \times 10^{-6}$ sec

at top of atmosphere: height $h = 10$ km
muons born with speed $v = 0.9999c$

Do they reach the ground?
Earth bystander: moving μ needs travel time
$t_{\text{moving}} = h/v = 33 \times 10^{-6}$ sec $> t_{\text{decay}} \rightarrow$ too long?!
should never see muons on ground! but we see plenty!
Why? muon "feels" at rest, and trip takes only
$t_{\text{rest}} = \sqrt{1 - v^2/c^2}\ t_{\text{moving}} = 0.5 \times 10^{-6}$ sec $< t_{\text{decay}}$
$\Rightarrow$ muon survive trip due to time dilation!

Relativity and Lengths

Turn light clock on side, use as yardstick:
at rest, clock length L_{rest}
shine light, front-to back roundtrip
$\rightarrow$ travel time $t_{\text{rest}} = 2L_{\text{rest}}/c$

HW6: bystander times light pulse
finds clock length to be

$$L_{\text{moving}} = \sqrt{1 - \frac{v^2}{c^2}} L_{\text{rest}} \quad (1)$$

Q: lesson?

Length Contraction

⋆ moving yardsticks don't appear to have same length as yardsticks at rest

⋆ moving yardsticks appears shorter

⋆ moving objects appear shorter in direction of motion!

⋆ space depends on state of motion! not universal!

iClicker Poll: Special Relativity

Which of these can *all* observers agree on
regardless of their state of motion?

I. Justin Bieber is 17 years old

II. Chambana and Chicago are 130 miles apart

III. radio signals from spacecraft move at speed c

A I only

B II only

C III only

D all of I, II, and II

E none of I, II, and II

The Special Theory of Relativity

Einstein 1905: realized all of the above

Implications deep and wide
Newton's laws of motion built upon absolute space, time
$\Rightarrow$ no longer valid! have to be rebuilt
to respect principle of relativity

Einstein did all of this as well in 1905
Revamped Newton's laws of motion $\rightarrow$ special relativity

- describes motions at any speed
- includes all wierdo time, space effects
- tested many ways many times, always found to agree with experiment
- e.g., particle acclerators (like Fermilab) would not work if we did not use relativity!

Energy in Relativity

Einstein revised expressions for energy:
a particle of mass m with speed v has (total) energy

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

Enormously important formula

Q: what is E when $v = 0$? What does this mean?
Q: what is E when $v \ll c$ but not zero? What does this mean?
Q: what is E when $v \to c$? What does this mean?

Mass is Energy

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

- if $v = 0$, particle at rest
  yet $E = mc^2$ not zero!
  represents energy due to *existence* of particle
  i.e., just due to presence of mass!
- energy output if mass m totally converted
  to some other energy form
- can be enormous: $m_{\text{donut}}c^2 = 4 \times 10^{15}$ Joules $= 1$ Mton TNT

So: mass is a form of energy!

Q: so what prevents donuts from exploding?
Q: when and where is *mass converted to energy?*

Mass $\Leftrightarrow$ Energy Interconversion

Implies: can convert mass to energy, *and* energy to mass
in fact–we do this every day at Fermilab!

energy $\rightarrow$ matter `www: Fermilab collision event`
2 particles with huge KE $\rightarrow$ many particles

but: if mass stays as mass, energy remains "stored"
and harmless (but tasty: mmmmm, donuts...)

Kinetic Energy Generalized

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

- if $v \neq 0$, then $E > mc^2$:
  speeding up a particle gives it more energy
  extra amount above mc^2 is kinetic energy (Einstein's version)

- if $v \ll c$ but $v \neq 0$, can show (HW 6)

$$E \approx mc^2 + \frac{1}{2}mv^2 \tag{5}$$

$$= \text{(rest energy)} + \text{(Newtonian kinetic energy)} \tag{6}$$

- Einstein sez: particles have *same* KE as Newtonian
  if the motion speeds slow compared to c

Q: Why did this conclusion have *to be true?*

In general (haven't proven this, but can):
At slow speeds ($\ll c$):
Special Relativity $\rightarrow$ Galilean/Newtonian physics

Had to be true!
- recall: a theory has to explain *all* data
- Newton was wildly successful – explained all available data until new measurements involving fast speeds (i.e., light)
- new theory (Special Relativity) must explain new data but also must explain all old data!

$\Rightarrow$ so SR must agree with Newton where Newton was successful

The beauty and power of Relativity:
★ *does* give back Newtonian results at slow speeds
★ explains conditions when Newton does and does not work!
★ provides larger, more complete picture!

Cosmology theory will have to respect these ideas of causality!

The Futility of Acceleration to Lightspeed

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (7)$$

What if $v \rightarrow c$?
• find $E \rightarrow \infty$

Interpretation:
can speed up a mass m, but as speed gets close to c...
• to get particle to go a little faster
takes more and more energy
• and to get particle to go at c requires *infinite* energy
physically unachievable, impossible!
• matter (objects with mass) cannot reach speed of light

Q: what if massless particle, $m = 0$*?*

Solve E equation for v:

$$v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \tag{8}$$

if $m = 0$, then $v = c$!

- to move at c, must be massless!
- light particles (photons) are massless!

technical note:

- more general formula works for all particles with mass m and momentum p:

$$E = \sqrt{(mc^2)^2 + (cp)^2} \tag{9}$$

- massive particles have relativistic momentum $p = mv/\sqrt{1 - v^2/c^2}$
- massless particles have $E = cp$
- all particles have $v/c = cp/E \leq 1$

The Cosmic Speed Limit and Causality

all particles (massive or massless) have $v/c \leq 1$
i.e., always have $v \leq c$

speed of light is universal speed limit

particles & information cannot travel faster than c
profound implications for cause & effect ("**causality**")

- an event can only affect future happenings which can be reached by light signal from the event
- so "region of future influence" *by* an event is defined by the (future) light cone of the event
- spacetime regions beyond the reach of a light signal cannot be affected!

Q: what portion of past *spacetime can affect an event?*