Astro 350
Lecture 20
Oct. 10, 2011

Announcements:
- HW6 due Friday
- Discussion Question 6 due Wednesday

Last time:relativity & dynamics, energy
- *Q: what does* $E = mc^2$ *mean?*
- *Q: what's the cosmic speed limit? why?*

- massive particles have $v = c\sqrt{1 - (mc^2/E)^2} < c$
  takes infinite energy to accelerate to c
  $\rightarrow$ never even reach lightspeed (sorry, Han!)
- massless particles (like photons) *always* move at c
  never faster or slower

speed of light is universal speed limit

Reported neutrino speed $v > c$ would contradict this limit
$\rightarrow$ a big deal if true!
as of today: 77 papers on this result in 16 days

This result aside, if c is universal speed limit:
Q: what part of spacetime can *be affected by an event?*

The Cosmic Speed Limit and Causality

all particles (massive or massless) have $v/c \leq 1$
i.e., always have $v \leq c$

speed of light is universal speed limit

particles & information cannot travel faster than c
profound implications for cause & effect ("**causality**")

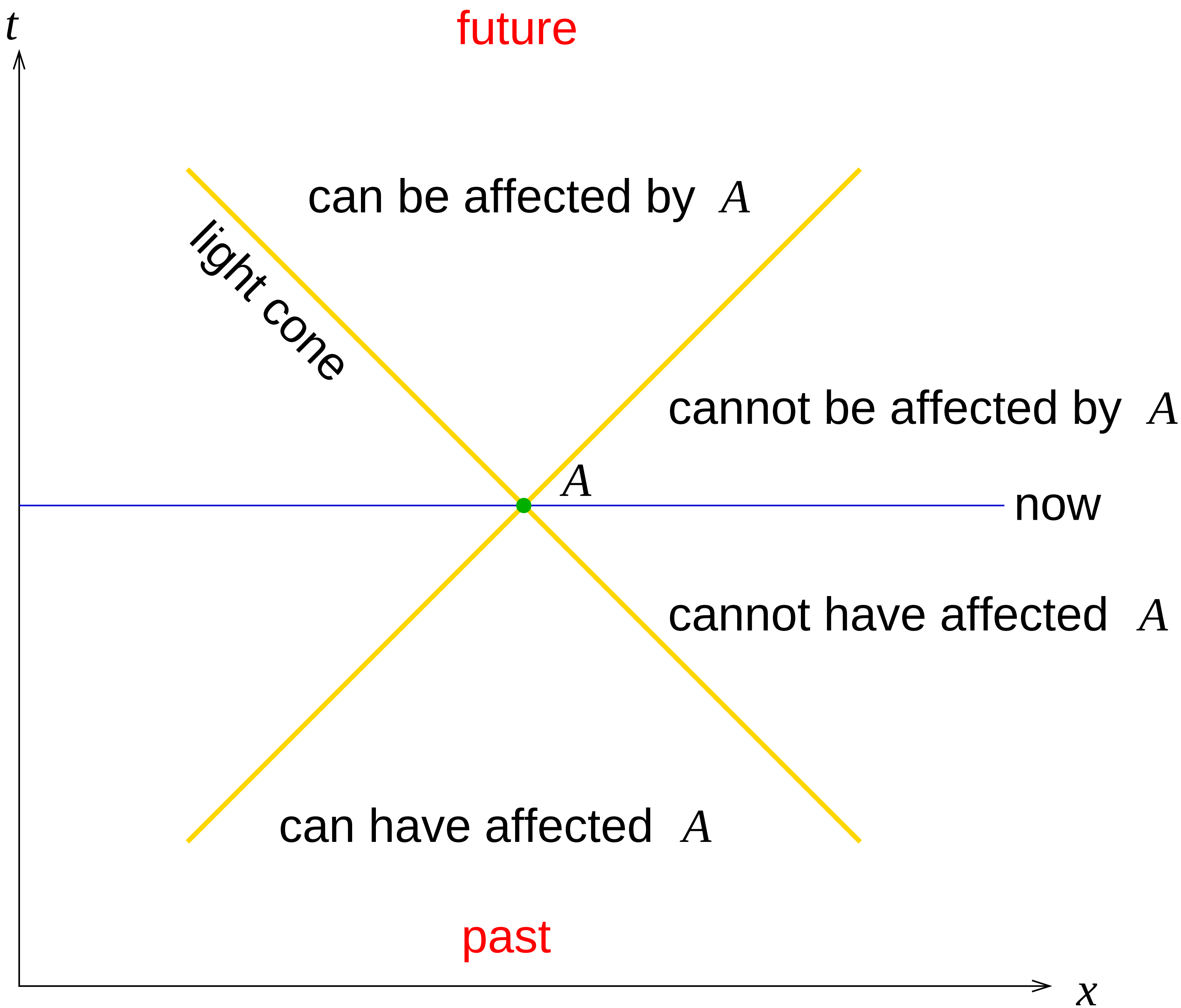
- an event can only affect future happenings which can be reached by light signal from the event
- so "region of future influence" *by* an event is defined by the (future) light cone of the event
- spacetime regions beyond the reach of a light signal cannot be affected!

Q: what portion of past *spacetime can affect an event?*

An event can only be affected by past events
which could have sent light signals to it
$\rightarrow$ so "region of past influence" *on* an event is defined by
the past light cone of the event
• light cones indicate causal regions related to events!

What's more: events outside light cone are
unaffected by the event!
Q: specifically, what's an event not *affected by your finger snap here and now?*

$\Rightarrow$ key Relativity result/outlook:
• information cannot travel instantaneously
• actions are "local" in the sense that
effects transmitted over finite distance in finite time


t
future
can be affected by A
light cone
cannot be affected by A
A
now
cannot have affected A
can have affected A
past
x

Extra for the Technorati: Invariants and the Interval

Different observers (typically) disagree about space, time
Is there *anything* they *do* agree about? yes!

Recall: relativistic $\vec{p}$, $E = \sqrt{(mc^2)^2 + (c|\vec{p}|)^2}$: observer-dependent
but: $(mc^2)^2 = E^2 - (c|\vec{p}|)^2$ same for everyone
$\rightarrow$ "invariant" quantity! everyone agrees on its value!

Another key example: two events and their "distance"

- *no* general agreement on separation in time Δt or in space $\Delta \vec{x}$
- *but* everyone agree on the value of $(\Delta s)^2 \equiv (c\Delta t)^2 - |\Delta \vec{x}|^2$, the "interval"

relativity built on relationships among invariant quantities
and how to connect these to experiences of observers
see today's Director's Cut extras for example

Special Relativity Executive Summary

⋆ Special Relativity:
includes high-speed motions (near c), doesn't include gravity

⋆ Space & Time
apparent distances, time intervals, simultaneity
not universal but depend on relative motion

⋆ Energy & Mass
can be converted into each other, mass is form of energy

⋆ Cause & effect ("causality")

- information cannot travel instantaneously
- actions are "local" in the sense that
  effects transmitted over finite distance in finite time

What About Gravity?

Special relativity beautifully accomodates light
(and all of electricity & magnetism)
but ignores gravity

How to include? consider Newton gravity force law

$$F_{\text{grav}} = \frac{GMm}{R^2} \tag{1}$$

gravity force due to mass M depends on distance R
and spreads over all space ($F \neq 0$ for any $R < \infty$)

but this implies that if M moves $\rightarrow$ R changes
$\rightarrow$ gravity force changes instantaneously over all space!
Einstein sez: this is totally illegal! an unmitigated disaster!
∞ no signal–including gravity–can move faster than c!

Big Al concludes: major changes needed!

The Equivalence Principle Revisited

How to go about revising gravity? Where to start?

Recall Galileo atop the Tower of Pisa:
gravity $\rightarrow$ all objects move (accelerate) the same way in free fall
regardless of object mass, shape, composition not new result,
but different explanations...

Newton sez:
it just so happens that gravitational mass
the way objects "feel" or "couple to" gravity $F_{\text{grav}} = m_{\text{grav}} g$
is always exactly the same as inertial mass
the way objects resist acceleration $a = F/m_{\text{inert}}$

Einstein sez:
too amazing to be a coincidence, must be deeper...

Einstein's Equivalence Principle

Einstein notes:
Gravity causes acceleration, but in "democratic" way:
all objects accelerate the same

Einstein's Equivalence Principle:
in a closed room, no experiment can distinguish
(non-gravitational) acceleration vs gravity
(note similar "feel" to Relativity Principle)

But note: acceleration is aspect of motion
relates to objects' travel through space and time
$\rightarrow$ gravity=acceleration equivalence will have impact
(i.e., bizarreness) on space and time

Experiments Inside an Accelerating Rocket

Consider a rocket in otherwise empty space

- that is, no gravity!
- moving with constant acceleration a

Experiment:
Astronaut Bart, standing on floor of rocket, has flashlight
holds it at height h, points horizontally, shines towards wall

iClicker Poll: Light Beam in Accelerating Rocket

in rocket with constant acceleration

Bart hold flashlight at height h, shoots beam horizontally

At what height will beam hit opposite wall?

A at same h

B higher than h

C lower than h

hint: easier to think about when looking at experiment
from non-accelerating viewpoint

key ideas:

light takes time to move across spaceship

during which, spaceship accelerates → gains v, moves vertically

in non-accelerating frame, see that

- light path is straight (horizontal) line
- spaceship vertical motion → far wall moved higher

⇒ light hits below where aimed

in accelerating frame (i.e., according to Bart):

agrees that light hits below where aimed, and concludes

★ light ray deflected

★ entire light path bent (in fact, a parabola!)

Q: but what does this mean, according to Al's Equiv Principle?

Gravitational Lensing

In accelerating spaceship: light rays bent

But by equivalence principle:
must find same result due to gravity, so:

⋆ gravity bends light rays
⋆ light "falls" too!
⋆ gravitating objects "attract" light rays
  distort light paths differently depending on
  how strong the gravity over each path

gravitating objects distort passing light
leads to distorted images of objects behind gravity sources

gravitational lensing

- observable effect, and in fact
- an increasingly powerful tool!

Accelerating Rockets & Clocks

consider "light clocks" installed in spaceship
- manufactured identically in Switzerland
- each emits light pulse every Δt microseconds

clocks, astronauts stationed in cieling (Milhouse) and floor (Bart) (height difference Δh)
Q: if rocket not *accelerating, do M & B see the other's clock tick at same rate as his own?*

Now fire rockets $\rightarrow$ spaceship has constant acceleration a

Compared to non-acceleration light travel time
Q: does the downgoing flash take longer/shorter/same time?
Q: does the upgoing flash take longer/shorter/same time?
Q: and by the equivalence principle...?

Time Warp: Gravitational Time Dilation

Clocks in accelerating spaceship:
Bart (floor observer) accelerating towards downgoing light
sees it sooner than if $a = 0$
B sez M's clocks running fast
Milhouse (cieling) accelerating away from upgoing light ray
sees it later than if $a = 0$
M sez B's clocks running slow

But equivalence principle says: gravity must do same thing! So...

★ clocks in basement appear to run slower
than clocks in attic!
in fact, attic clocks appear faster by amount
$\Delta t = t_{\text{attic}} - t_{\text{basement}} = g\Delta h^2/c^3$ a tiny effect unless g huge
★ time "warping" but now due to gravity:
"**gravitational time dilation**"
★ gravity influences "flow" of time!

Director's Cut Extras

For the Technorati: More on the Interval

for two nearby events: different observers with different motions

- *disagree* on event separations in time Δt and space Δx
- but *agree* on **interval** Δs
  which each observer calculates from his/her Δt, Δx:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 \tag{2}$$

Example: observer A snaps fingers twice
all while bystander B sees A move at speed v

Interval according to A
all observers perceive self at rest
$\rightarrow$ $(\Delta x)_A = 0$ and $(\Delta t)_A =$ time between snaps, and
calculates interval $(\Delta s)_A = c(\Delta t)_A$
$\rightarrow$ interval is $c\times$ time diff for observer located at both events!

Interval according to B

bystander B sees A moving at speed v

$\rightarrow$ in time interval $(\Delta t)_B$ sees A move dist $(\Delta x)_B = v(\Delta t)_B$

calculates interval

$$(\Delta s)_B = \sqrt{(c\Delta t)_B^2 - (\Delta x)_B^2} = (c\Delta t)_B \sqrt{1 - \frac{(\Delta x)_B^2}{(c\Delta t)_B^2}} = (c\Delta t)_B \sqrt{1 - \frac{v^2}{c^2}} \tag{3}$$

Invariance

But interval is invariant, so $(\Delta s)_A = (\Delta s)_B$ and thus

$$(\Delta t)_{\text{rest}} = (\Delta t)_A = (\Delta t)_B \sqrt{1 - \frac{v^2}{c^2}} = (\Delta t)_{\text{moving}} \sqrt{1 - \frac{v^2}{c^2}} \tag{4}$$

we recover the time dilation formula!