Astro 350

Lecture 22

Oct. 14, 2011

Announcements:

- HW6 due now
- Discussion Question 7 due next Wednesday
- HW7 available, due in 1 week

Last time: gravitational lensing

Q: what is it?

Q: why is it a great tool for cosmology?

Lensing by Dark Matter in Other Galaxies

If background galaxy (or quasar) light passes thru
foreground galaxy or galaxy cluster
can resolve lensed arcs of background object `www: arcs`
use to reconstruct total mass distribution of foreground gal
$\Rightarrow$ direct probe of dark matter distribution!

Status: already done for tens of objects
conclude: total gravitating mass $\gg$ visible mass
$\rightarrow$ independent evidence for dark matter!
not only that, but can infer DM distribution!
`www: map of DM in cluster`

The Bullet Cluster and Dark Matter

Bullet cluster:
two galaxy clusters in process of merging
have already passed through each other(!) once *Q: how?*

Can observe:

- optical galaxies
- X-rays: hot gas that filled cluster interiors before merger
  in fact: more (ordinary) matter in gas than in galaxies!
- lensing $\rightarrow$ all gravitating mass, so dark matter = total - seen

Results:

- X-rays (hot gas) offset from galaxies
  since stars don't collide with each other, but gas "splatters"

iClicker Poll: Bullet Cluster

Bullet cluster:
most of *visible mass* in splattered hot gas
offset from non-colliding *galaxies*

Where will lensing reveal *total* mass?

A if weakly interating DM: with gas
if gravity modified: with gas

B if weakly interating DM: with galaxies
if gravity modified: with gas

C if weakly interating DM: with gas
if gravity modified: with galaxies

D if weakly interating DM: with galaxies
if gravity modified: with galaxies

If weakly interacting DM:

doesn't collide with anything

$\rightarrow$ acts like stars in galaxy

should be seen with galaxies

If no dark matter but modified gravity

most gravity where most ordinary matter:

$\rightarrow$ should see gravity source with intracluster gas

`www: Bullet Cluster`

lensing data $\rightarrow$ gravitation source centered on galaxies

$\rightarrow$ consistent with weakly interacting dark matter

$\rightarrow$ not consistent with alternative gravity!

The General Theory of Relativity

1915: Einstein publishes General Theory of Relativity
a.k.a. **General Relativity**, a.k.a. GR
landmark intellectual achievement

keeps all key concepts from Special Relativity
- no absolute space, time
- light always moves at c, matter $< c$
- mass is form of energy
- causality: no particles, signals, info travel $> c$

but now fully includes gravity: GR is the modern theory of gravity

Key GR Idea I:
equiv principle $\rightarrow$ gravity affects all objects the same
$\rightarrow$ gravity is not a force but a property of space & time!
but gravity source is matter, so:
GR is theory connecting matter, space, and time!

If gravity isn't a force, what is it?
hint: already saw that gravity "warps" time

Key GR Idea II:
according to GR, gravity is "curvature" of space & time!?!
i.e., gravity can "warps" both space and time
$\rightarrow$ spacetime "curved"

- gravitational redshifting, time dilation, light bending are all manifestations of this
- curved orbits of particles due to gravity in GR are really responses to spacetime curvature!
- note: gravity = geometry! harkens back to Greeks!

GR Slogans (T-Shirt/bumper sticker/text msg):

- matter tells spacetime how to curve
- curvature tells matter how to move

these ideas are beautiful and powerful
but also not (for most people) intuitive or trivial

best way to learn is from examples
will focus on two key examples of relativistic spacetimes

- example #2: the Universe
  rest of the course after today
- today: example #1....

Interlude: Refresher on Energy

Recall: objects moving only due to gravity

total energy

$$E_{\text{tot}} = \text{kinetic} + \text{potential} = \frac{1}{2}mv^2 - \frac{GMm}{R} \tag{1}$$

Special case: object at large distance $\to \infty$, at rest $v = 0$

falls to position R

- *Q: What is* E_{tot} *at first?*
- *Q: how does KE change? PE?* E_{tot}*?*
- *Q: to launch object from* R *to* ∞*, what* v *needed?*

Escape Speed

in example: initially, $KE = 0$ and $PE = 0$ so $E_{\text{tot}} = 0$
and E cons $\rightarrow$ $E_{\text{tot}} = 0$ always

- as falls, speeds up $\rightarrow$ KE$\uparrow$ but
  gets closer $\rightarrow$ $R\downarrow$ $\rightarrow$ $PE\downarrow$ (more negative)
- so always true that $0 = \frac{1}{2}mv^2 - GMm/R$
  which means $v^2 = 2GM/R$, or $v = \sqrt{2GM/R}$

★ gives fall speed v at distance R
  e.g., $E = 0$ fall from ∞ to Earth surface:
  hit with speed $v = 25,000$ mph! `www: terrestrial craters`

to launch object from Earth $\rightarrow$ at rest at ∞:
need final state to have $E_{\text{tot}} = 0$, so
always need $E = 0$ $\rightarrow$ same case!
$\rightarrow$ need launch with $v_{\text{esc}} = \sqrt{2GM/R}$ **escape speed**

Q: what if launch with $v > v_{\text{esc}}$? with $v < v_{\text{esc}}$?

if launch with $v = v_{\rm esc}$, then
$E_{\rm tot} = 0$, and $KE = PE$ always
particle escapes, but at rest when far away ("∞")

If launch with $v > v_{\rm esc}$ then $KE > PE$
$\rightarrow$ when far away, $PE \rightarrow 0$ but still $KE \neq 0$
$\rightarrow$ particle escapes, still moving when far away
$\rightarrow$ $E_{\rm tot} = KE + PE > 0$ $\rightarrow$ particle "unbound"

If launch with $v < v_{\rm esc}$ then $KE < PE$
$\rightarrow$ at some distance, slow down to $v = 0$ $\rightarrow$ $KE = 0$
but still $PE < 0$ $\rightarrow$ can't go farther
$\rightarrow$ turn around, fall back: particle "bound"
$\rightarrow$ $E_{\rm tot} < 0$: not enough energy to escape

Can predict the future: just ask what is $v_{\rm launch}$ vs $v_{\rm esc}$?
- if $v_{\rm launch} < v_{\rm esc}$: object falls back (pop fly)
- if $v_{\rm launch} \geq v_{\rm esc}$: object escapes (rocket)

Black Holes

Laplace (1790's): escape velocity $v_{\rm esc} = \sqrt{2GM/R}$
What if star has mass M, and $R < 2GM/c^2$?
then $v_{\rm esc} > c$!
light cannot escape! $\rightarrow$ black hole

Wrong argument (Newtonian gravitation) ...but right answer!

General relativity predicts existence of black holes
and their properties

Black Hole Properties

any object of *any* mass M can (in principle) become a black hole!

size: Schwarzchild radius

$$R_{\rm Sch} = \frac{2GM}{c^2} \qquad (2)$$

radius also provides BH "recipe":

- crush object M smaller than $R_{\rm Sch}$ $\rightarrow$ get BH!
- example: for mass of Sun $R_{\rm Sch} = 2GM_\odot/c^2 = 3.0$ km
  but actual $R_\odot = 7 \times 10^6$ km
  $\rightarrow$ the Sun is not a black hole! (whew!)
- for mass of Earth: $R_{\rm Sch} = 1$ cm!

The Black Hole Horizon

Why call R_{Sch} the BH radius? nothing is there!

True, but: R_{Sch} marks "point of no return"
horizon: surface enclosing the BH
i.e., horizon is surface of sphere w/ radius R_{Sch}

horizon is one-way "membrane"
once inside $r \leq R_{\text{Sch}}$ nothing can escape...even light!
cosmic roach motel!

Hence:
no light escapes $\rightarrow$ black
but nothing else moves as fast $\rightarrow$ nothing else escapes $\rightarrow$ hole