Astro 350
Lecture 27
Oct. 26, 2011

Announcements:
- Good news: no HW this week
- Bad news: Hour Exam 2 in class next time

FYI: History's most prolific planet hunter here today!
Dr. Bill Borucki, head of NASA *Kepler* planet finder
"Kepler Mission: An Overview of Science Results"
Physics Colloquium, Loomis 141, 3:30 pm

Kepler status:
- 2000 planet candidates discovered
- many earth-sized
- some in habitable zone (liquid water)

$\Rightarrow$ will discover the first Earth-like habitable world!

Last time: lifestyles in an expanding universe
cosmic scale

key quantity: scale factor – measures "photocopy enlargement"

$$\ell(t) = \quad a(t) \quad \times \quad \ell_0$$

$\ell(t)$	$a(t)$	ℓ_0
AB distance at t	scale factor	present AB distance
time varying	*time varying*	*fixed once and for all*

So: entire history of the universe
contained in the details of
how scale factor a grows with time!

- lengths grow as $\ell(t) \propto a(t)$
- areas grow as $A(t) \propto a(t)^2$
- volumes grow as $V(t) \propto a(t)^3$

Here endeth the material on Hour Exam 2

iClicker Poll: A Pop Fly

A ball is launched upwards from the Earth's surface

What will happen later?

A it will eventually fall back down

B it will leave earth and never return

C either (a) or (b), depending on launch speed

Cosmodynamics II

$a(t)$ gives expansion history of the Universe
but: *How does scale factor $a(t)$ grow with time?*

Cosmic Evolution: Intuition

Ballpark analogy: a pop fly
Q: what are possible fates?
Q: what theory tools would we use to find these fate?
Q: what factors influence which occurs?
Q: how would we predict which will occur?

The Universe is a pop fly!
Given current dynamic state of the Universe *Q: which is...*
Q: what are possible future outcomes?
Q: what factors influence which occurs?
Q: how would we predict which will occur?

Gravity & Fate: Baseball vs Cosmology

Pop fly: upgoing ball in Earth's gravity field; possible fates:
(1) fall back
(2a) leave Earth; $v \neq 0$ at infinity
(2b) leave Earth; $v = 0 \rightarrow$ "barely escape"

Factors:
gravity (downward)
vs inertia (upward)

How predict? Pop Fly
gravity $\rightarrow$ escape speed $v_{\text{esc}} = \sqrt{2GM/R}$
inertia $\rightarrow$ launch speed v_0
$\rightarrow$ fate set by ratio v_{esc}/v_0

What about the Universe?
same ideas! Gravity vs inertia!
will find similar key: gravity/inertia ratio

Cosmic Evolution: Quantitative Analysis

full description: comes from General Relativity
quick 'n dirty: Non-relativistic (Newtonian) cosmology

at time t, pick arbitrary point as origin $\vec{R} = 0$,
enclose in arbitrary sphere of radius $R(t)$:
diagram: sphere, R

enclosed mass $M(R) = 4\pi/3 \; R^3 \rho = const$
consider a small "test" mass m on edge of sphere
"feels" gravity due to sphere mass
Q: what is Newtonian acceleration of test mass?

Newtonian Cosmodynamics

Newton says:

a mass m accelerates due to force: $m \times \text{accel} = F$

if force due to gravity–free fall–then $F = GM(R)m/R^2$

and so acceleration is

$$m\ddot{R} = -\frac{G\ M(R)m}{R^2} \tag{1}$$

where $-$ sign reminds us gravity is *attractive Q: how*?

but note–"test" mass cancels (equivalence principle), so

$$\ddot{R} = -\frac{G\ M(R)m}{R^2} = -\frac{4\pi}{3}G\rho R \tag{2}$$

Newton sez:

$$\ddot{R} = -\frac{4\pi}{3} G \rho R \tag{3}$$

Hubble & Einstein say:

Universe is expanding, so sphere radius
moves according to scale factor: $R(t) = a(t)\ R_0$

$$\ddot{a} R_0 = -\frac{4\pi}{3} G \left(\rho + 3\frac{P}{c^2} \right) a R_0 \tag{4}$$

$$\ddot{a} = -\frac{4\pi}{3} G \left(\rho + 3\frac{P}{c^2} \right) a \tag{5}$$

- R_0 cancels! scale factor accel indep of sphere size! had to be this way $\rightarrow$ cosmo principle
- Einstein adds term with pressure P

Q: what is Newtonian energy of test mass?

Newtonian Cosmodynamics II: Energy

test mass m at edge of gravitating sphere has energy

$$\text{kinetic} + \text{potential} = \text{total} \quad (6)$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = E \quad (7)$$

solve for speed v:

$$v^2 = 2\frac{GM}{R} + 2\frac{E}{m} \quad (8)$$

$$= \frac{8\pi}{3}G\rho R^2 + 2\frac{E}{m} \quad (9)$$

Q: what do Hubble and Einstein say about v?

Newton says:

$$v^2 = \frac{8\pi}{3} G\rho R^2 + \frac{2E}{m} \tag{10}$$

Hubble and Einstein say:
speed $v = HR = \frac{\dot{a}}{a} R$, so

$$H^2 R^2 = \dot{R}^2 \;=\; \frac{4\pi}{3} G\rho R^2 + \frac{2E}{m} \tag{11}$$

expansion technology: $R(t) = a(t) R_0$

$$H^2 a^2 = \dot{a}^2 = \frac{4\pi}{3} G\rho a^2 - K \tag{12}$$

The Friedmann Equations

Friedmann Acceleration Equation

$$\text{cosmic acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{13}$$

important features:

- *Q: significance of $-$ sign?*

Friedmann Equation ("Energy Eq.")

$$(\text{cosmic expansion rate})^2 = H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \tag{14}$$

where K is a constant

- *Q: how does expansion rate depend on contents of U?*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{15}$$

note $-$ sign:

- due to attractive nature of gravity
- galaxy gravity on each other slows expansion

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \tag{16}$$

- for any time t, relates expansion rate $H(t)$ = change in a to constant K and values of $\rho(t), a(t)$ at t
- cosmic contents (density) influences expansion
- K term can be important – or zero!
  value, sign of constant K has to be measured

Q: Friedmann gives history of U–how? what ingredients needed?

Hour Exam 2

`www: Exam Info Online`

Sample Questions: Multiple Choice

• In a future industrial accident, the Sun is collapsed to a black hole, with the same mass as it has today. What is the effect on the Earth's orbit?
(a) the Earth is pulled into the black hole
(b) the Earth is pulled close to but not inside the hole
(d) the Earth is driven out of the Solar System
(c) the Earth's orbit is unaffected

• Two galaxies move relative to us according to Hubble's law. If the galaxies show the same redshift, then
(a) they *must* lie at the same distance from us
(b) they *cannot* lie at the same distance from us
(c) they *may* lie at the same distance from us

Sample Questions: Short Answer

- *The Principle of Relativity*

(a) What is the Principle of Relativity? Explain in 1-2 sentences.
(b) Give one example of an effect that is a consequence of this principle.

- *Black holes, dark matter, and gravitational lensing*.

(a) Give two properties of black holes that make them good dark matter candidates.
(b) Briefly explain what is gravitational lensing?
(c) Briefly explain how can we use gravitational lensing to detect black holes in our Galaxy's halo.
(d) Searches as described in part (c) show that our Galactic halo contains few, if any, black holes. Briefly (1-2 sentences) explain why it is unlikely that black holes are the dark matter in the halos of other Galaxies.