Astro 350
Lecture 28
Oct. 31, 2011

Announcements:

- Discussion Question 8 due Wednesady
- HW 8 due Friday
  Hints for Q2: in *static* U, $a(t) = \text{constant} = 1$ always!
  and $\rho =$ constant always and everywhere

Recall: lifestyles in an expanding universe

- cosmic expansion: all distances grow as $r(t) = r_0 a(t)$
  $\rightarrow a(t)$ encodes cosmic expansion history
- but how do we know what $a(t)$ is?

Friedmann equations

$$\text{cosmic acceleration} \;=\; \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{1}$$

$$(\text{cosmic expansion rate})^2 \;=\; H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \tag{2}$$

Solving for the History of the Universe

Friedmann gives expansion rate

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{K}{a^2} \quad (3)$$

in terms of cosmic ingredients

Strategy:

⋆ know scale factor $a(t_0) = 1$ today

⋆ Friedmann tells how a changes (that's H)

⋆ use Friedmann to "run movie backwards/forwards"
and find $a(t)$ for all times t
but this *is* enough to work out cosmic history
e.g., temperature, densities at all cosmic times

Q: what information needed to solve for H?

Friedmann Ingredients:

1. Need (constant) value of K
1. Need (total) cosmic density

$$\rho_{\rm total} = \rho_{\rm ordinarymatter} + \rho_{\rm dark\ matter} + \rho_{\rm radiation} + \cdots (?) \qquad (4)$$

$\rightarrow$ need cosmic ingredients, and how density of each changes with cosmic scale factor

Densities

matter = stuff that moves slowly compared to c

non-relativistic: $v \ll c$, $kT \ll mc^2$

if not matter produced/destroyed *Q: how* could *it be created/destroyed?*

then mass density $\rho = M/V \propto 1/V \propto a^{-3}$

so *matter density* $\boxed{\rho_{\mathrm{matter}} = \rho_{\mathrm{matter},0} a^{-3}}$

radiation = stuff moving at or near speed c

in extras today: show that

radiation density $\boxed{\rho_{\mathrm{rad}} = \rho_{\mathrm{rad},0} a^{-4}}$

Cosmic Crystal Ball: Solving Friedmann

Fried. $\rightarrow a(t)$ evolution
but need to know how ρ depends on a
$\rightarrow$ the evolution U. depends on what's in it!

Simplified "toy model" – matter-only universe:
when $\rho \approx \rho_{\text{matter}} = \rho_{\text{ordinary matter}} + \rho_{\text{dark matter}}$
includes (and is mostly!) dark matter
and $K = 0$ (or ρ term $\gg K$ term):
"matter dominated U"

Solution method (for technorati): $H \propto \sqrt{\rho} \rightarrow \dot{a}/a \propto a^{-3/2}$
$a^{1/2} da \propto dt \;\; \Rightarrow \;\; a^{3/2} \propto t$ and $a \propto t^{2/3}$
and we always put today's value $a(t_0) = 1$, so

$$\boxed{a_{\text{matter-dom}}(t) = (t/t_0)^{2/3}}$$

Huge! Gives (a possible) history of universe!

A Matter-Only Universe

in this model (and only this model): $\boxed{a(t) = (t/t_0)^{2/3}}$

in this model:

Q: What is the fate of the universe? what happens as $t \to \infty$?

- can solve $H(t) = \frac{\dot{a}}{a} = \frac{2}{3}\frac{1}{t}$

  Hubble "constant" changes with time!

Q: what is behavior of H over time?

Q: what does this imply for evolution of the U?

Q: what does this mean physically?

matter-only fate: $a \propto t^{2/3}$, so after a long time
when $r \to \infty$, then $a \to \infty$: expand forever!

but: expansion slowing $\to$ U. decelerating
gals: outward momentum opposed by inward gravity

evaluate matter-only expansion *today*

$$H_0 = \frac{2}{3}\frac{1}{t_0} \quad \text{and so} \quad t_0 = \frac{2}{3}\frac{1}{H_0} = \frac{2}{3} t_{\text{Hubble}} \tag{5}$$

predict t_0 from H_0: "expansion age"
related to Hubble time–but not equal to it

We already know $t_{\text{Hub}} = 13.7$ billion years $= 13.7$ Gyr
• which gives expansion age $t_0 = 9.1$ Gyr
yet we also know: oldest stars (globular clusters)
have $\sim 12 - 14$ Gyr age
Q: what's going on?

More to the Picture Than Meets the Eye

What just happened?

We assumed we had a universe only made of *matter*
a *"what you see is what you get"* universe

but this makes many testable predictions
- including age of U vs Hubble const
- measured both ofthese, found matter-only prediction false!

So: we *do not live* in a matter-only universe!?!
Top on cosmologist's to-do list: figure out what's going on!

∞

Q: how?

Studying the History of Cosmic Expansion

Recall: Friedmann sez that at any cosmic "moment"
("epoch"− measured by time t and/or redshift z)
the cosmic expansion H directly related to
cosmic contents ρ

So: imagine we could measure expansion rate H
at many different epochs, finding $H(z)$ at many z

this determines the cosmic expansion history
then Friedmann $\rightarrow$ density history, e.g, $\rho(z)$

$\star$ cosmic motion $\leftrightarrow$ cosmic contents
$\star$ expansion measures cosmic density!
$\star$ can compare with expectations if $\rho = \rho_{\text{matter}} + \rho_{\text{radiation}}$

Expansion Archaeology

Goal: measure expansion rate at past times

Strategy:

recall Hubble's law $v(t) = H(t)\ r(t)$: always true
but as you look at distant objects
light travel time becomes large ("time machine effect")
$\rightarrow$ no longer probing expansion today,
  when rate is $H(t = t_0) = H(z = 0) = H_0$
  but rather expansion in past $H(\text{high} - z)$
$\Rightarrow$ Can use this to get expansion history
  key requirement: need distance r to high-z objects

Q: what techniques are available?

Supernovae and Cosmodynamics

goal: measure expansion out to high z

key tool: **standard candle** $\equiv$ object of known L
measure flux F, then $d_{\rm lum} = \sqrt{F/4\pi L}$

need objects which:

- have fixed L indep of z, environment
- can see at high z $\rightarrow$ high L $\rightarrow$ supernova explosions

- Massive stars $\rightarrow$ SN: Type II
  bright, but: L varies w/ mass, metallicity X bad idea!
- SN Type Ia: exploding white dwarf
  WD $\rightarrow$ fixed mass of ^{56}Ni (radioactive) $\rightarrow$ ^{56}Fe
  decay sets $L(t)$ $\rightarrow$ std candle!
  `www: SN 1994D`

iClicker Poll: Past Expansion Rate

in the real universe, with gravity

How should the expansion rate change with time?

A expansion slows with time $\rightarrow$ faster in past

B expansion constant with time $\rightarrow$ same in past

C expansion speeds up with time $\rightarrow$ slower in past

SN Ia and Expansion History

SN Data: distance indicator ($d_{\text{lum}}(z)$ "luminosity distance")
traces expansion history $H(z) = \dot{a}/a$

- but expansion $H = \dfrac{\text{rate of change in } a}{a}$: a kind of "velocity"
  so variation in $H \rightarrow$ change in "velocity"
- H history measures cosmic acceleration

expectations: acceleration $= \ddot{a}/a = -\dfrac{4\pi G}{3}\left(\rho + 3\dfrac{P}{c^2}\right)$
$\Rightarrow \ddot{a} < 0$ gravity slows expansion

Pop fly Analogy

galaxies moving apart today $\leftrightarrow$ upgoing ball
would coast forever, but gravity attractive
slowing galaxies $\leftrightarrow$ slowing ball
$\rightarrow$ faster in past

Distant Supernovae: The Verdict

SN data:

$H(z)$ **smaller** in the past (high z and small t)

$\Rightarrow H(z)$ **increases** with time!

$\Rightarrow \ddot{a} > 0$!

accelerating expansion!

`www: SN Ia data`

2011 Nobel Prize in Physics

given to Saul Perlmutter, Brian Schmidt, and Adam Riess
`www: 2011 Nobel Prize`

> for the discovery of the accelerating expansion of the Universe through observations of distant supernovae

Q: why is this Nobel-worthy?

Q: notice what the prize does not *mention?*
that is, what does their work not *tell us?*

An Accelerating Universe: Implications

Recall: expected deceleration because ordinary matter
(even dark matter!) has gravitational attraction
But: found acceleration → something present which
has gravitational repulsion! "antigravity"!?!
...and enough of it to overwhelm the attraction
of ordinary matter!

Pop Fly Analogy

Universe is accelerating
→ galaxies faster as they move apart
so upgoing ball would speed up, zoom away from view!!

In more detail: SN Ia: $\ddot{a} > 0$, but Friedmann sez

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right)$$

$\Rightarrow \rho + 3P/c^2 < 0$

$\Rightarrow P < -3\rho c^2$ negative pressure!?!

Physical "interpretation":

recall: $F =$ pressure $\times$ area *diagram: piston,* A, P, F

$P > 0$: outward force (e.g., ideal gas)

$P < 0$: inward force (e.g., elastic)

Director's Cut Extras

Radiation Density and Cosmic Expansion

radiation = stuff moving at or near speed c

relativistic: $v \sim c$, $kT >> mc^2$

if particles not created/destroyed,

then total number const

- number density $n = N/V \propto a^{-3}$

energy density $\varepsilon_\gamma = E/V = E_\gamma n_\gamma$

where E_γ is the average energy of *one* particle

but we saw for photons: $E_\gamma \propto 1/a$

$\rightarrow \varepsilon_\gamma \propto a^{-4} \propto T^4$ if thermal but mass-energy equivalence $E = mc^2$ says that

$$m = \frac{E}{c^2} \tag{6}$$

$$\frac{m}{V} = \frac{E}{V}\frac{1}{c^2} \tag{7}$$

$$\rho = \frac{\varepsilon}{c^2} \tag{8}$$

and so

$$\boxed{\rho_{\mathrm{rad}} = \varepsilon_{\mathrm{rad}}/c^2 \propto a^{-4}} \tag{9}$$