Astro 350

Lecture 6

Sept. 2, 2011

Announcements:

- HW1 due now
- HW2 available, due in 1 week
- Discussion Question 2 posted, due next Wednesday

Last time:

- Galileo: special cases of motion–free bodies, free fall
  *Q: what are they defined as? what are the motions like?*
- Newton's Laws of Motion
  *Q: Laws I and II?*
  *Q: how does Newton II enable fortunetelling and archæology*
  *Q: what information do you need to predict future & past?*

# Fortunetelling (and Archæology!) with Newton II

*input*: at initial time, need to know/specify

- object mass $m$
- all of forces acting on $m$

$\Rightarrow$ find *net force* $F$

*turn the math crank*: $a = F/m$

$\rightarrow$ find *acceleration* = change in velocity

$\rightarrow$ use this to find new position, new velocity
at at moment a little later

$\rightarrow$ at new time and position, find new net force

...lather rinse repeat

Result: find particle path in future!

But also: can mathematically "run the move backwards"
and predict the past history as well!

## Newton III: "Action-Rection"

a rule about how forces behave
between two objects

if 2 bodies interact:
the **force** exerted by object 1 on object 2
  is **equal and opposite** to
the **force** exerted by object 2 on object 1

*Q: application–you standing still*
*Q: Jump shot*

# Explaining Kepler

Kepler I: planets move in ellipse
this is curved path
direction of motion changing

So: velocity changes
$\rightarrow$ planets **accelerate**

$\Rightarrow$ need force
**gravity**

*diagram: Sun, planet*
label force free motion, deflection due to gravity, actual motion

# Universal Gravitation

Newton's Theory: combined all of the following ideas

- gravity acts beyond earth
- gravity directed on line connecting centers of bodies
- gravity strength decreases with distance
- all bodies are sources of gravity
  $\Rightarrow$ everything attracts everything else

Can summarize mathematically compact way:
for 2 bodies, masses $m_1$, $m_2$
centers separated by distance $R$

gravitational force:

$$F \propto m_1$$
$$F \propto m_2$$
$$F \propto 1/R^2$$

together:

$$\boxed{F = G\frac{m_1 m_2}{R^2}} \tag{1}$$

where $G$ is just a fixed, constant number, same always:

$$G = 6.7 \times 10^{-11} \ \frac{\text{m}^3}{\text{kg s}^2} \tag{2}$$

- *Q: how is equation similar/different from list on previous slide?*
- gravity force $F \propto 1/R^2$ : "inverse square law"
  *Q: force on satellite* $2\times$ *as far from earth center as us?*

satellite $2\times$ as far from earth center as we are
$\Rightarrow$ Force is $1/2^2 = 1/4$ as strong, i.e., 4 times weaker

why? $F(R) \propto 1/R^2$ for any $R$
so: compare at $R = R_e$ (earth radius) and $R = 2R_e$

proportional means that

$$\frac{F(2R_e)}{F(R_e)} = \frac{1/(2R_e)^2}{1/R_e^2} = \frac{1/(4R_e^2)}{1/R_e^2} = \frac{R_e^2}{4R_e^2} = \frac{1}{4} \tag{3}$$

# Gravity and Planet Motion

Newton II: input is force, output is motion
For planets around Sun, force is gravity (*free fall!*)

So: What is motion when $F = G\frac{m_{Sun}m_{planet}}{R^2}$?
Now just a math problem.
*diagram: sun, planet orbit,* $\vec{v}$, $\vec{F}$

Newton II + Gravity: properties of predicted orbits

- orbit is ellipse, with sun at one focus
- equal areas in equal times
- $a^3_{\text{AU}} = P^2_{\text{yr}}$
  actually better, more info: $a^3 = GM_{\text{Sun}}P^2/4\pi^2$
  *Q: why is this better?*

So: *Newton's laws + gravity force* $\rightarrow$ *Kepler's laws !*
*theory agrees with observation!*

*Q: effect on planets of Sun's mass doubled?*
*Q: effect on planets of Sun's size doubled?*

Note: only force on planet is gravity: free fall

$$m_{\text{planet}} a_{\text{planet}} = F_{\text{planet}} = G\frac{m_{\text{planet}} M_{\text{Sun}}}{R^2} \quad (4)$$

$$a_{\text{planet}} = G\frac{M_{\text{Sun}}}{R^2} \quad (5)$$

free fall acceleration only depends on Sun mass $M_{\text{Sun}}$
and Sun-planet orbit distance $R$

$\rightarrow$ **independent** of planet mass or size!

$\rightarrow$ at same $R$, all objects accelerate same way

$\Rightarrow$ equivalence principle pops out of Newton gravity! Woo hoo!

"Turning the Dials"

- double Sun's mass $\rightarrow$ double acceleration $\rightarrow$ faster orbits
- double Sun's size $\rightarrow$ same mass $\rightarrow$ no change in orbits

# Testing Newton's Gravity

Moons of Jupiter: orbits obey Kepler's laws
$\rightarrow$ Jupiter's gravity works like Sun's, Earth's

1830's: Uranus observed orbit did *not* follow predictions
of Newtonian solar system model
*the death Newton's gravity?*

Remember: have to agree with **all** data, not just some
even one clear failure is enough to kill theory
e.g., Kepler and Mars: just a small discrepancy from circular
but still had to throw out circular orbits

maybe...but also: maybe have not included all sources of gravity
maybe unknown object causes U's deviations
$\Rightarrow$ a new planet?

# iClicker Poll: Uranus Discrepancy

1830's Problem: *measured* Uranus orbit doesn't match predictions of Newtonian Gravity *theory*

Vote your conscience!

Which seems more likely to you?

A Newton's gravity theory *correct*, but not all gravity sources had been included

B Newton's gravity theory *incorrect* (or at least incomplete)

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*Q: what experiment/observation would tell which is right?*

if unknown object, could predict where should be
did this, looked, and in 1846...
Neptune found at right position—*predicted* by Newton's gravity

other more recent tests:

Binary Stars: two stars orbiting each other
move in ellipses, obey Kepler's laws
`www: binary star orbit`
$\rightarrow$ Gravity theory works outside solar system

Kepler described planet motion
Newton explained plant motion

- result from nature of gravity
- a complete, accurate, powerful theory

## Copernican Revolution

the Earth is a typical planet
one among many that orbit the Sun
not the center of the solar system

...only the first of many such revolutions!