

**Astronomy 350 Fall 2012**  
**Homework #1**

Due at the start of class Friday, Sept. 7

*Note:* Your homework solutions should be legible and include all calculations, diagrams, and explanations. You are encouraged to type the short essay responses. The TA is not responsible for deciphering unreadable or illegible problem sets! Also, quantitative problems are graded on the method of solution, not just the final answer; you may not get any credit if you just state the final answer!

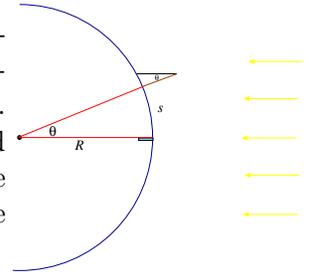
1. *The Geocentric Cosmology.*

- (a) **[5 points]** Make the strongest scientific case for a geocentric view of the solar system. Be sure to mention observations and the way they the geocentric model explains them.
- (b) **[5 points]** How would you make a scientific argument *against* the geocentric model to someone in the year Y0K? That is, build your argument based only on naked-eye observations, not any based on telescopes, spacecraft, airplanes, or other modern technology.

2. *A Gut Feeling for Astronomical Numbers [5 points].* We will see that the observable part of the universe has a size of roughly  $10^{23}$  kilometers (km), and the age of the universe is about 14 billion years =  $1.4 \times 10^{10}$  years. Such enormous timescales are hard to grasp in an intuitive way. To begin to develop a feel at least for the large numbers, we will scale the problem down in considering large numbers of seconds rather than years.

- *Estimate* (don't look up!) the width of a single piece of paper and express your answer in meters (*hint:* think for example about how thick a book is). If the universe were  $10^{23}$  *pages* across, how big would it be? Compare your answer to the size of the Chambana, and of the Earth (diameter about 12,000 km), and comment.
- Consider the time span of 1 million seconds =  $10^6$  sec. Rewrite this in a more convenient familiar unit—i.e., hours, days, months, years, or centuries. Will a semester of ASTR350 take more or less than a million seconds? (In real time—not how it seems!)
- How many million seconds are in a year?
- Now consider 1 billion seconds =  $10^9$  sec. Do you know anybody this old? What historically was going on  $10^9$  sec ago?
- If the universe were 14 billion *seconds* old, how old would that be, in more familiar units (years, centuries)? Replacing each second with a year, you perhaps begin to get an idea of the tremendous (but finite) age of the universe.

3. *Eratosthenes and the Size of the Earth* The ancient Greek cosmologist Eratosthenes was able to show that the Earth is round and to accurately measure its radius  $R$ . Eratosthenes lived in present-day Egypt, around 200 BC. He noticed that on the longest day of the year, the Sun was straight overhead in the town on Syene. But on the same day, in the city of Alexandria, the Sun cast shadows at an angle  $\theta = 7^\circ$  from the vertical. You may treat the Sun's rays as parallel. See diagram at right.

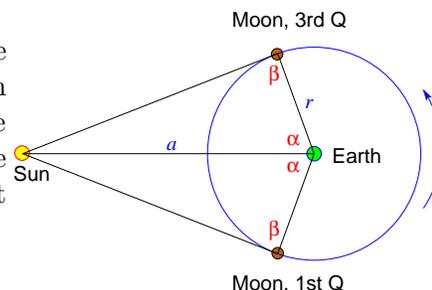


- (a) [5 points] What do we learn about the Earth from the simple fact that the Sun is seen at two different angles at two different places when viewed at the same time?
- (b) [5 points]. Let's call the distance between Syene and Alexandria  $s$ ; this is the length of an arc of angle  $\theta$ . Using properties of arcs of circles, find an equation relating the radius  $R$  to the arc length  $s$  and angle  $\theta$ .<sup>1</sup> In this expression, substitute the measured value for  $\theta$  and the distance  $s = 800$  km to find  $R$ . Compare your result to the actual value for  $R = 6400$  km, and comment.
4. *Baseball and the Scale of the Solar System* [10 points]. To get a gut feeling for the relative scales of the Solar System, imagine that Dr. Evil creates a diabolical shrink ray which zaps the solar system, reducing the size of everything in it by the same proportion. The new size of the Sun (normal radius  $R_{\text{Sun}} = 7 \times 10^5$  km) is now that of an ordinary baseball ( $r = 1.5$  inches = 4 cm). In this "shrunk" solar system, find the radius of the proportionally shrink-rayed Earth (ordinary radius  $R_{\text{Earth}} = 6400$  km). Draw to scale a sketch of this shrunk Earth (you may also sketch Dr. Evil if you so choose). Include the Sun on your drawing, with the Sun-Earth distance also to scale; but if you cannot do this, indicate why, and indicate where the Sun should be placed if you had a large enough piece of paper. Finally, the distance to the nearest star is about  $d = 1.3$  parsec =  $2.6 \times 10^5$  AU. If the baseball Sun were on the pitchers' mound of Illini Field, about where would the nearest shrunk star be? How far would the farthest star in our shrunk Galaxy be (real distance about 20,000 pc = 20 kpc)?
5. *The Motion and Layout of the Solar System* [10 points]. **Choose one** of the following.
- (a) These questions give a gut feeling for Kepler's laws.
- What would our lives be like if Earth had a highly eccentric orbit around the Sun?
  - Consider objects whose orbits lie *entirely* within that of the Earth (i.e., never reaches 1 AU). Can any such object have an orbital period  $> 1$  year? If so, give an example of such an orbit. If not, explain why not.
  - Now consider objects which have *part* of their orbit inside 1 AU and part outside of 1 AU. Can any such object have a period  $P > 1$  year? If so, give an example of such an orbit. If not, why not? Same thing for  $P < 1$  yr.

<sup>1</sup>*Hint:* if you don't recall the equation, it may help to notice that for an arc that is really the whole circle, we have  $\theta = 360^\circ$  when the arc length is the circumference  $s = 2\pi R$ . Then think of that  $s$  and  $\theta$  are when we have a semicircle, and then a quarter of a circle. Do you see the pattern?

- The most massive solar system object beyond Neptune is not Pluto, but a body discovered three years ago, known as Eris (And now, officially, neither are planets). Eris has a semimajor axis  $a = 68$  AU. How many times will the Earth go around the Sun when Eris does so once?

(b) *For the more technically adventurous:* The Greek astronomer Aristarchus proposed an ingenious method for determining the relative scale of the Earth-Moon orbit relative to the Earth-Sun orbit. Consider the diagram at right.



- The first quarter moon is the phase when exactly  $1/2$  of the moon is illuminated—the righthand side. The third quarter moon is the phase when exactly  $1/2$  of the moon is illuminate—but now the lefthand side. Explain why this means that the angle  $\beta = 90^\circ$ , and thus the upper and lower triangles are similar and are right triangles.
- A bit of thought will show that the angle  $\alpha < 90^\circ$ . It turns out that the best way to measure  $\alpha$  is to measure the two time intervals,  $t_{1Q \rightarrow 3Q}$  from first to third quarter moons, and  $t_{3Q \rightarrow 1Q}$  from third to first quarter moons. Assuming the Moon moves in a circular orbit at constant speed, find an expression for  $\alpha$  (in degrees or in radians) given these two time intervals. *Hint:* the moon's period for one complete orbit must be  $P = t_{1Q \rightarrow 3Q} + t_{3Q \rightarrow 1Q}$ .
- Now that we know how to get  $\alpha$ , find an expression that uses only  $\alpha$  to find the ratio  $r/a$ , where  $r$  is the Earth-Moon orbit radius and  $a = 1$  AU is the Earth-Sun orbit radius. This then gives the Moon's orbit radius in AU, so that we can draw the above diagram to scale (which it is not!).
- *For style points:* Let's turning the problem around: find the actual values of  $r$  and  $a$ . Use these to calculate the value for  $\alpha$ . How difficult will this be to measure based on naked eye observations?

## 6. Newton's Laws

**[5 points].** You and a friend are in a broken-down spacecraft which floats at rest, waiting for the interstellar towtruck to arrive; you both are strapped into your seats for safety. The towtruck is late and you are bored, so to pass the time, you throw a basketball to your friend, who catches it, all the while with both of you remaining strapped in your seats. You may ignore any effects of air resistance.

- While the ball is in flight between you, will your spacecraft be moving? If not, why not? If so, how will the spacecraft's direction of motion compare to the ball's direction, and will the spacecraft move slower or faster than the ball?
- After your friend has caught the ball, will the spacecraft be moving? If not, why not? If so, how will the spacecraft's direction of motion compare to the ball's direction, and will the spacecraft move slower or faster than the ball?