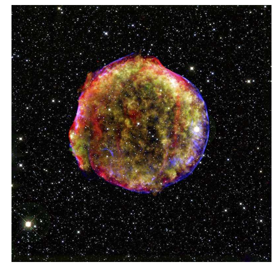
Astronomy 350 Fall 2012
Homework #2

Due in class: Friday, Sept. 14

1. [**5 points**]. *Supernova Archæology* As discussed in class, the great astronomer Tycho Brahe in the year 1572 discovered a "new star" which in fact a supernova, the explosion of a dying massive star. This was one of the closest supernova explosions in recorded history. This object is known as SN 1572 or Tycho's supernova, and long ago faded from view in visible light. But the supernova remains bright in X-rays, which were used to make the image at right, taken by the NASA's *Chandra* X-ray telescope in the year 2003.



   The measured radius of the supernova is 8 parsec, where 1 parsec $= 3.1 \times 10^{13}$ km. The age of the supernova is just the time between the Tycho's discovery and the *Chandra* observation. Using these, find the speed v at which the supernova is expanding, in km/sec. Comment on your answer. Also compare your answer to the speed of light, $c = 3.0 \times 10^5$ km/s.

2. *Universal Gravity and You.*

   (a) [**5 points**]. Calculate the *ratio* $F_{\rm Sun}/F_{\rm Earth}$ of the gravitational force $F_{\rm Sun}$ of the Sun on you to that of the force $F_{\rm Earth}$ of the Earth on you. Note that the Sun has mass 2.0×10^{30} kg and is at a distance of $1\,\mathrm{AU} = 1.5 \times 10^8$ km, while the Earth has mass 6.0×10^{24} kg and radius 6400 km.

   (b) [**5 points**]. Comment on

      i. How much does the Sun affect your weight?

      ii. Without calculating anything, comment on how much the other planets in the solar system are likely to affect your weight.

      iii. Comment on whether your result is reasonable in light of the fact that we don't seem to be sucked off the Earth's surface into the Sun.

      iv. Imagine the Sun had a bigger affect on your weight. How would this be noticeable in your daily life?

   (c) [**5 points**]. The nearest star, called alpha Centauri (or αCen) is about 1.3 parsecs away. The star has nearly the same mass as the Sun, so for simplicity assume it has the same mass as the Sun. Calculate the *ratio* $F_{\alpha{\rm Cen}}/F_{\rm Earth}$ of the gravitational force $F_{\alpha{\rm Cen}}$ of αCen you to that of the force $F_{\rm Earth}$ of the Earth on you.

   Comment on the influence of nearby stars on your weight.

3. *Alternative Universes: A New Gravity Law* [**5 points**]. In class we saw that Newtonian gravity provides an incredibly successful theory of motion in the solar system and in many other astronomical systems To get an appreciation for how Newton's

gravity affects our lives, let's imagine a universe with a different gravity law. In this "bizarro" universe, the gravity force between two masses (call them m_1 and m_2) is still attractive, is still directed along a line between them, and is still proportional to each of the masses, and still depends on the distance R between the two masses.

In our real universe, Newton's gravity force has an inverse-square dependence on distance: $F_{\rm grav,us} = G_{\rm us} m_1 m_2 / R^2$. However, in the alternate bizarro universe, the gravity force depends *directly* on the distance, so that the gravity force is $F_{\rm grav,bizarre} = G_{\rm bizarre} m_1 m_2 R$, where $G_{\rm bizarre}$ a constant number measured by the bizarro universe cosmologists. What are some ways in which the bizarro universe be different from ours? Do not do any calculations, but think about what would happen as a result of the difference in the gravity force law. Hint: you might think about what (if anything) would be different for planets, the Sun, the solar system, or our Galaxy).

4. *How far can the eye see?* The unaided human eye can see objects down to a minimum brightness (flux), which we will call $F_{\rm min}$. This give the minimum apparent brightness the naked eye can detect. It turns out that a star with the same luminosity as the sun (call this luminosity $L_{\rm sun}$) at a distance of $d = 1$ parsec will have a flux of about $F = 300 F_{\rm min}$, i.e., it will have a flux 300 times higher than this minimum.

   (a) [**5 points**]. Find the maximum distance (call it $d_{\rm max}$) at which a Sun-like star would still be visible to you naked eye. *Hint:* it is useful to find the ratio of the flux at $d_{\rm max}$ to the flux at $d = 1$ pc. Express your answer in pc.

   Compare you answer to the distance to the nearest star, and comment.

   Also compare you answer to the size of the Galaxy, and comment.

   (b) [**5 points**]. Some stars die in supernova explosions, when they can briefly have a luminosity of $L = 10^{10} L_{\rm sun}$. Find the maximum distance at which such an explosion can be seen by your naked eye. Express your answer in pc.

   Compare you answer to the size of the Galaxy, and comment. Also compare your answer to the distance to the nearest galaxy, about 10^6 pc, and comment on why supernova explosions are very useful for cosmologists.

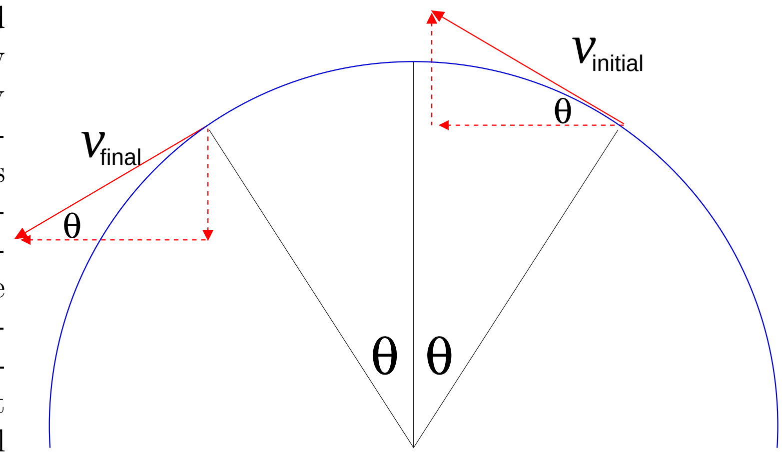
5. *Circular Motion and Acceleration.* In class it was asserted that an object in circular orbit around a spherical gravitating body has an orbit radius R and speed v that together give the gravitating mass

$$M = \frac{v^2 R}{G} \tag{1}$$

   Here we will show how this comes about.

   (a) [**5 points**]. An important aspect of the problem is the nature of circular motion. In particular, circular motion involved constant acceleration, directed towards the center, with magnitude $a_{\rm circ} = v^2/R$. Do **one** of the following options.

Option 1: In the diagram at right, we see the velocity at two instants (solid arrows). The dashed arrows show the component of velocity in the horizontal and vertical directions. Show that the *change* in velocity in the x (horizontal) direction is zero, while there is a nonzero change in the velocity in the y (vertical) direction, and that the change is directed towards the center. Note: for circular motion at constant speed v, it follows that while the initial and final velocities are different (different directions), the initial and final speeds (velocity "length" v) are the same.



Option 2 (for the technorati): Show that for circular motion, $\vec{a} = v^2/r\ \hat{r}$. One approach is use the kinetic energy in polar coordinates, $T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$ along with the Euler-Lagrange equations. Another is to use the Cartesian vector equation $\vec{r}(t) = [x(t), y(t)] = [r\cos(\omega t), r\sin(\omega t)]$ where $r = |\vec{r}| = \sqrt{x^2 + y^2} = const$, along with the definitions $\vec{v} = \dot{\vec{r}}$ and $\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$.

(b) [**5 points**]. Since the motion is due to gravity, the acceleration $a_{\text{circ}} = v^2/R$. has to be due to the gravitational force. If this is the case, show that $M = v^2R/G$.

(c) [**5 points**]. Explain how equation (1) allows us to learn about dark matter in galaxies. That is, what do we measure and how does this lead us to conclude that there is dark matter?