Astronomy 350 Fall 2011
Homework #4

Due in class: Friday, Oct. 5

1. [**5 points**]. High in the evening summer sky, three stars are quite bright: Deneb, Altair, and Vega. These together form a triangle, known as the Summer triangle. Do **one** of the following problems.

   (a) The ratio of fluxes for Vega and Deneb is $F_{\rm Vega}/F_{\rm Deneb} = 3.1$. These stars have parallaxes $p_{\rm Vega} = 0.13$ arc sec, $p_{\rm Deneb} = 0.0023$ arc sec. Show which star is closer to us. Then show which star has a higher luminosity.

   (b) *For the Technorati.* Stellar brightnesses are measured and tabulated in the "magnitude" scale. A magnitude is a logarithmic measure of flux, defined so that a magnitude *difference* is *ratio* of fluxes:

   $$m_2 - m_1 = -2.5 \log_{10} \frac{F_2}{F_1} \quad (1)$$

   with unfortunate prefactor of 2.5, and even more unfortunate $-$ sign, which means that large magnitudes correspond to faint objects. In the Summer triangle, the stars have magnitudes $m_{\rm Altair} = 0.77$, $m_{\rm Deneb} = 1.26$, and $m_{\rm Vega} = 0.04$. Their parallaxes are $p_{\rm Altair} = 0.20$ arc sec, $p_{\rm Deneb} = 0.0023$ arc sec, and $p_{\rm Vega} = 0.13$ arc sec. Use these data to place the stars in order of luminosity, and find the ratio $L_{\rm Altair} : L_{\rm Deneb} : L_{\rm Vega}$.

2. *The Rise of the Heavy Elements.* We will soon see that the elements hydrogen and helium were created in the early universe–in fact, during the first seconds of the big bang, But the early universe created no heavier elements ("metals") at all. On the other hand, at its birth the solar system did contain heavy elements, though still mostly hydrogen and helium. The table below summarizes the composition of both composition of the early universe and of the solar system at birth,[1] giving percentages of ordinary matter in each form.

Element	Abundance in Early Universe	Abundance at Solar Birth
hydrogen (H)	75%	70%
helium (He)	25%	28%
heavier elements ("metals")	0%	2%

As discussed in class, elements heavier than helium are made deep inside stars. When stars end their lives, they eject some of their insides into space. The solar system was born after many generations of stars lived and died.

   (a) [**5 points**]. Explain how we can understand how the abundances (percentages) of hydrogen, helium, and heavy elements *each* have *changed* between the big bang and the birth of the solar system. That is, why would we expect *each* of these to go up or down as they do?

[1] Note that in fact, this is the composition of the solar system when it was formed; we are not concerned here with the helium that has been produced since by the Sun in the Sun's core.

(b) [**5 points**]. For *each* of hydrogen, helium, and heavy elements, sketch a graph of how its abundance has changed over time from the big bang to the birth of the solar system. That is, make three plots of abundance versus time, and for example the heavy element plot should have a time axis going from the big bang to solar birth, and the abundance axis should go from 0% to 2% heavy elements. All you have been given are the beginning and ending abundances for each element, so sketch your best guess as to how the abundances changed over the times in between.

(c) [**5 points**] Briefly explain how the composition of a star can be deduced from the star's spectrum. What part of the star has the composition we measure?

Note that nuclear reactions in stars only occur in their cores, and do not affect the star's outer layers. Thus the star's outer layers retain the original composition of the star, until the star dies.

In light of these facts, what do we learn when we measure a star's composition?

(d) [**5 points**]. Observations of stars in our own Galaxy tell us that Milky Way stars have heavy element abundances spanning wide range. Compared to other stars, the Sun is somewhat more metal-rich than average. Some stars are found to be very metal poor. A very rare few have been found with metal abundances 100,000 times lower than that of the Sun (that is, heavy elements comprise 0.00002% of the observable portion of the star).

Based on your answers to parts (b) and (c), what is the reason that some stars have very low heavy element abundances? Why are these stars considered very interesting?

What would you predict would be the helium abundances measured in these low-metal stars?

Finally, for a few such stars is possible to estimate the star's age. Do you expect them to be younger than the Sun, older, or about the same age? Explain.

(e) [**5 points**]. The most distant galaxies we see far across the universe generally have very low abundances of heavy elements. On the other hand, nearby galaxies have heavy element abundances similar to those in our Galaxy and in the Sun. Explain how we can understand this trend. *Hint*: remember we are using light to make these observations.

3. *Neutrinos from the Sun.*

(a) [**5 points**]. What is a neutrino? Under what conditions are neutrinos produced? What important lesson(s) do we learn from the fact that we observe neutrinos coming from the Sun?

(b) [**5 points**]. Neutrinos are weakly interacting (but luckily *not* non-interacting!). Thus neutrinos produced in the Sun have a small but not zero chance of colliding with your body. Specifically, for each second of time, each kilogram in your body has a probability of about $p = 3 \times 10^{-12}$ that a high-energy solar neutrino will interact with it. That is, the odds are 3 in 10^{12}, or three in one trillion, that such an interaction will happening in 1 kg over a timespan of 1 sec.

The probability that a neutrino will interact with more than 1 kilogram in one second is just p times the number of kilograms. The probability that a neutrino will interact with 1 kilogram over more than 1 second is just p times the number of seconds. Thus the number N_{int} of neutrino interactions in a mass M_{kg} kilograms in a time t_{sec} seconds is just

$$N_{\text{int}} = pM_{\text{kg}}t_{\text{sec}} \tag{2}$$

Using this expression, find the number of neutrino interactions in the body of a $M = 100$ kg person over a lifetime of 100 years. Hint: you will need to find out how many seconds are in 100 years.

Comment on how much we should be worried about being irradiated by solar neutrinos during our lifetimes.

(c) [**5 points**]. Experiments to detect solar neutrinos are designed to detect at least one neutrino a day, i.e., $N_{\text{int}} = 1$ during a time of 1 day. Using eq. (2), find the mass needed to for the detector. Express your answer in kg and in metric tons, where 1 metric ton is 1000 kg. Hint: you will need the number of seconds in a day.

Comment on why solar neutrino experiments are so large.

(d) [**5 bonus points**]. Our calculations have so far ignored the fact that half the time, it is night. How will this affect the results in neutrino experiments?

(e) [**5 points**]. Neutrinos are an excellent candidate for dark matter. Explain why neutrinos meet the two basic requirements to be dark matter.