Astronomy 350 Fall 2012
Homework #5
Due in class: Friday, Oct. 12

1. *A Gut Feeling for Dark Matter*

   (a) [**5 points**]. Mass density is defined as $\rho = M/V$, the ratio of mass to volume. At the Sun's $R = 8$ kpc distance from the Galactic center, the dark matter mass enclosed in a sphere of radius R is about $M = 10^{11} M_\odot$. Using this, find the mass density of dark matter $\rho_{\rm dm}$ at the location of the Earth. Express your answer in kg/m^3. Compare your result to the density of liquid water, which is $\rho_{\rm water} = 1000$ kg/m^3.

   (b) [**5 points**]. If dark matter takes the form of elementary particles, as many cosmologist think, current theories favor an individual particle mass of $m_{\rm dm} \approx 100 m_{\rm proton} = 1.6 \times 10^{-25}$ kg. If this is the case, find the *number* of dark matter particles per cubic meter at the location of the Earth.

   (c) [**5 points**]. Now make a rough estimate the volume of $V_{\rm me}$ your body. You are welcome to make the "cylindrical human" approximation, or even the more drastic "spherical human" approximation–a rough ballpark number is all that is needed here, not any sort of ultrapersonal disclosure! Express your answer in cubic meters (m^3).

   (d) [**5 points**]. Finally, combine the results from part (b) and part (c) to estimate the number of dark matter particles in your body at any given time. Comment on your result.

2. *Particle Dark Matter*

   (a) [**5 points**]. If dark matter is in the form of elementary particles, it has to be weakly interacting. To see this, imagine the contrary–that dark matter could readily collide with ordinary matter. Explain what Earth-based and solar system experiments could be done to rule out such particles.

   (b) [**5 bonus points**]. Dark matter has to be electrically neutral. To see this, imagine that the DM is positively charged, with the same charge as a proton. How would an electron respond to such a particle? How would a dark matter + electron system reveal itself in lab experiments?

3. *Length Contraction.* In class we considered a light clock on its side. This a traincar whose length is $L_{\rm rest}$ when measured by an observer at rest in the traincar. A light pulse that travels from back to front and back again. And observer in the car times the roundtrip time of the light pulse, which we will call $t_{\rm rest}$. All the while, the train is moving at speed v relative to a bystander. The bystander measures the roundtrip time for the light pulse to be $t_{\rm moving}$.

   (a) [**5 points**]. Show that $t_{\rm rest} = 2L_{\rm rest}/c$.
   For the case of the bystander who sees the train as moving, notice that the light pulse appears to go faster than the train by speed $c - v$ when it goes back-to-front, and after reflection appears to move at speeed $c - v$ towards the back.

Using this information, show that

$$t_{\text{moving}} = \frac{L_{\text{moving}}}{c - v} + \frac{L_{\text{moving}}}{c + v} \tag{1}$$

where L_{moving} is the the apparent distance the light travels.

(b) [**4 points**]. Finally, use the results from part (a), as well as the time dilation formula $t_{\text{moving}} = t_{\text{rest}}/\sqrt{1 - v^2/c^2}$, to verify that $L_{\text{moving}} = \sqrt{1 - v^2/c^2}\ L_{\text{rest}}$.

4. *Special Relativity: Length Contraction and Time Dilation in the Real World.* Special relativity predicts strange effects when one observer measures the behavior of clocks and yardsticks of another observer in relative motion. These results are not effects we are aware of in everyday life; here we will see why.

In all parts of this problem, the situation is that there are two observers, A and B, in relative motion with constant velocity. Their relative speed is v and along the x-axis. You should ignore effects of gravity, i.e., this is a special relativity problem only. Each observer has a yardstick of length $L_{\text{rest}} = 1$ yard (as measured by the observer her/himself) which lies along the x-axis, and a clock that ticks once every $(\Delta t)_{\text{rest}} = 1$ second (as measured by the observer her/himself).

(a) [**4 points**]. Assign names to the observers A and B. Use these throughout the rest of the problem.

First, consider a relative speed v which is familiar from everyday experience, say $v = 65$ mph $= 30$ meters/sec. A observes B moving with this speed. Calculate the length L_{obs} and tick rate $(\Delta t)_{\text{obs}}$ that A observes B's instruments to have. Also calculate the same results that B observes A's instruments to have.

Note that your calculator may have trouble here, giving a perhaps surprising answer. If so, explain where the problem crops up.

(b) [**4 points**]. Now consider a faster speed: $v = 200$ km/sec (about the Sun's orbit speed around the center of the Galaxy). As in part (a), find A's impressions of B's instruments and vice versa.

At this much higher speed, how noticeable are the relativistic effects?

(c) [**4 points**]. Finally, consider a speed $v = 0.99\,c$. Again, find A's impressions of B's instruments and vice versa. Comment on your result.

(d) [**4 points**]. On the basis of your answers from parts (a)-(c), come up with a "rule of thumb" for when (i.e., at what speeds v) weird relativity effects are noticeable, and when they are not. Go on to explain why in everyday life we don't notice length contraction and time dilation.