Astronomy 350 Fall 2012
Homework #6
Due in class: Friday, Oct. 19

1. *Black Holes and Time Dilation.*

   (a) [**5 points**]. A giant clock is illuminated far from a black hole (i.e., $r \to \infty$). As the image of the ticking clock approaches the hole, brave observers hovering in rockets will see the tick duration change from $(\Delta t)_{\rm emit} = 1$ sec. What will the observed tick duration be at $r = 2R_{\rm Sch}$? at $r = 1.001R_{\rm Sch}$? at $r = R_{\rm Sch}$? Comment on your result.

   (b) [**5 points**]. As the astronauts lower themselves towards the black hole, what changes (if any) do they see in the tick rates of their own wristwatches?

2. *The Bullet Cluster and Dark Matter.* In class we discussed the "Bullet Cluster" of galaxies.

   (a) [**5 points**]. What is the bullet cluster? What do observations show about the different forms of matter inside the bullet cluster?

   (b) [**5 points**]. How does the bullet cluster give evidence in favor of dark matter and against modifications to gravity theory?

   (c) [**5 points**]. In your opinion, how much (if at all) does this strengthen the case for dark matter?

3. *Galaxy Maps and the Cosmological Principle.* As discussed in class, cosmologists have made increasingly larger and better maps of the universe, and will make larger maps in the future. This is done by surveying large regions of the sky, and measuring the galaxies found. The results are often presented as "pie slices" – across a long, narrow strip of the sky, each galaxy's angular position is plotted versus its distance (really, redshift z). The results for two slices of the Two-degree Field Galaxy Redshift Survey ("2dF") appear on the next page.

   The maps are drawn with our location at the center (where the pie tips meet), and each galaxy is shown as a dot. Note that the regions outside of the filled-in pie slices were not covered by the survey, so the absence of points does not mean a true lack of galaxies!

   (a) [**5 points**]. The two "pie slices" are maps of regions on opposite sides of the celestial sphere. Comment on how the two slices compare with each other in detail, and in their average properties. Explain how this comparison is important for the cosmological principle. Also state what part of the cosmological principle is verified by this comparison.

   (b) [**5 points**]. Looking at the regions inside 1.0 billion lightyears (lyr), we see that there are clumps with many more galaxies than average ("overdensities"). Estimate the size $r_{\rm clump,max}$ of the largest clumps you see–how large would be the radius of a circle that encloses the largest clump? Use the distance scale given in billions of light years to estimate $r_{\rm clump,max}$ in billions of lyr. Finally, find $r_{\rm clump,max}$ in Mpc, using the fact that 1 billion lyr = 310 Mpc.

(c) [**5 points**]. In the nearest 1.0 billion lyr of the map, there are also voids with many fewer galaxies than average ("underdensities"). Estimate the size $r_{\rm void,max}$ of the largest void you see–how large would be the radius of a circle that encloses the largest void? Express $r_{\rm void,max}$ in billions of lyr and in Mpc.

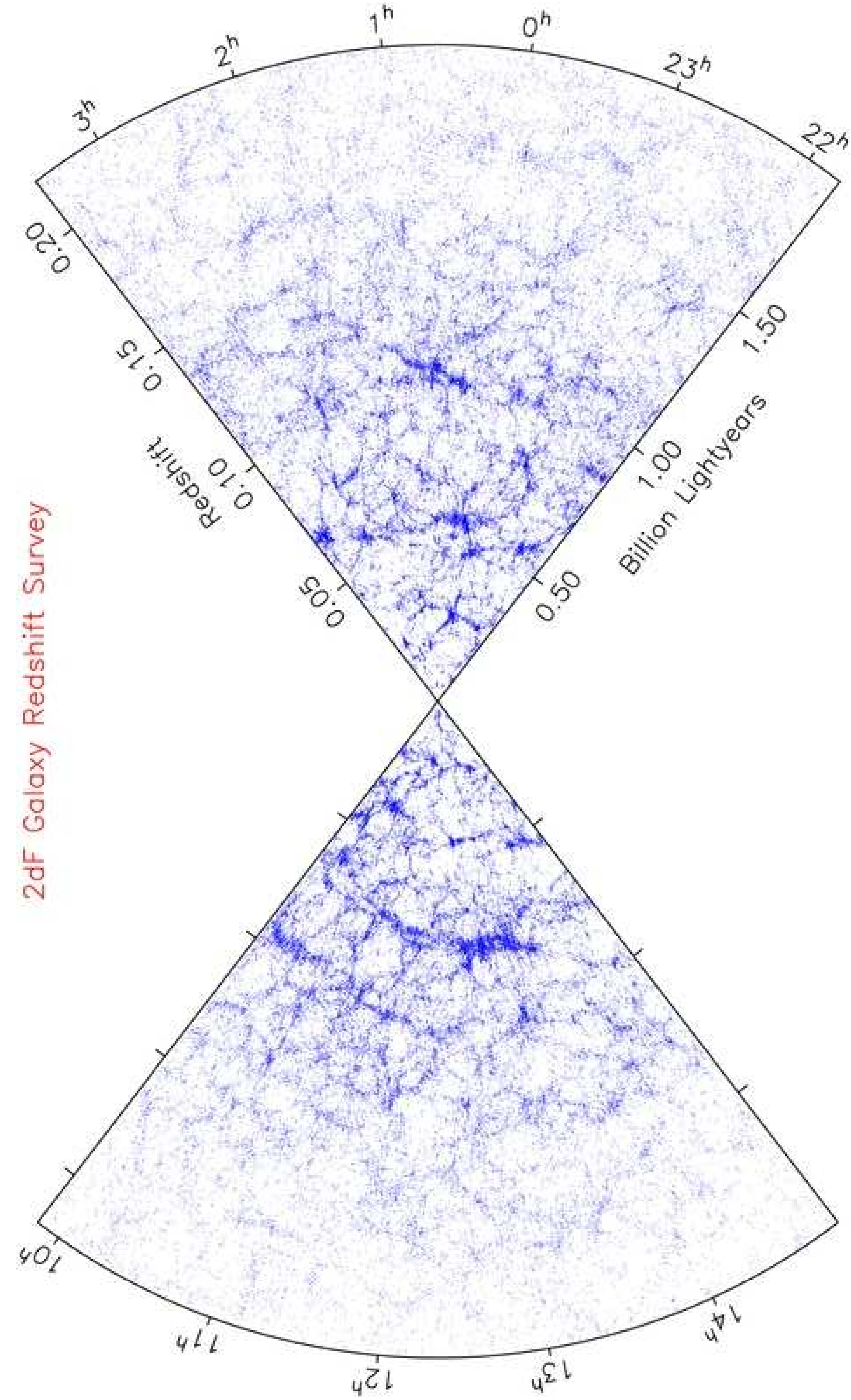
Finally, comment on how $r_{\rm void,max}$ and $r_{\rm clump,max}$ compare–which (if either) is larger?

(d) [**5 points**]. Imagine a circle with a radius *much smaller* than both $r_{\rm void,max}$ and $r_{\rm clump,max}$. Then imagine randomly drawing (or better, draw!) many such small circles in the inner 1.0 billion lyr region of the map, and counting the galaxies in each circle. Compare the numbers of galaxies found in the small circles–do these galaxy counts have *variations* that are large or small? How homogeneous is the universe on these small scales?

Then imagine a circle with a radius *much larger* than both $r_{\rm void,max}$ and $r_{\rm clump,max}$. Then imagine randomly drawing (or better, draw!) many such large circles in the inner 1.0 billion lyr region of the map, and counting the galaxies in each circle. Compare the numbers of galaxies found in the large circles–do these galaxy counts have *variations* that are large or small? How homogeneous is the universe on these large scales?

(e) [**5 points**]. Based on your answers to part (d), explain why $r_{\rm void,max}$ and $r_{\rm clump,max}$ roughly determine the length scale beyond which the universe becomes homogeneous.

(f) [**5 bonus points**]. Consider now the outer edges of the maps, i.e., the regions beyond 1.0 billion lyr that lie near the "pie crust." Here we see the density of galaxies detected by the survey becomes much smaller than in the inner region of the maps. Explain why this might *seem* to violate the cosmological principle. Then explain why this in reality does *not* violate the cosmological principle–*hint:* if galaxies nearby and far away have similar properties (e.g., luminosities), how will they appear different to the survey telescope?



Source: `http://magnum.anu.edu.au/%7eTDFgg/Public/Pics/2dFzcone%5fbig.jpg`