Astronomy 350 Fall 2012
Homework #8

Due in class: Friday, Nov. 9

1. *A Nobel Prize for Cosmology.* One year ago, the 2011 Nobel Prize in Physics was awarded to the cosmologists Saul Perlmutter, Brian Schmidt, and Adam Reiss, "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae."

   (a) [**5 points**]. How do observations of distant supernovae reveal that the Universe is accelerating?

   (b) [**5 points**]. Why is the discovery that the Universe is accelerating a result worthy of a Nobel Prize?

2. *A Static Universe and Einstein's Greatest Blunder(?)* Once Einstein invented General Relativity, he went on in 1917 to apply this theory to a homogeneous and isotropic universe (as he correctly guessed it ought to be, inventing the cosmological principle along the way). To his surprise and dismay, he found that General Relativity demands that the universe is *dynamic*. That is, Einstein's theory *demands* that the universe either expands or contracts, and *forbids* a "static" universe in which galaxies are (on average) at rest relative to each other.

   (a) [**5 points**]. What does it say about the idea of an expanding universe that even Einstein did not have the courage to stand by his own theory and go ahead and predict, in 1917, the cosmic expansion that Hubble discovered in 1929?

   (b) [**5 points**]. It is not too hard to see why General Relativity does not allow a static universe. Consider the two Friedmann equations, which tell how the scale factor changes with time:

$$\text{(cosmic acceleration)} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho \tag{1}$$

$$\text{(expansion rate)}^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \tag{2}$$

   (we have ignored pressure, which complicates the equation but won't affect any of our conclusions). First explain why a static universe must have both zero acceleration and zero expansion rate. Then using the Friedmann equations, show that requiring both expansion and acceleration rates to be zero leads to a contradiction with the fact that the universe is not totally empty. Finally, use the fact that the universe isn't totally empty to argue that it must therefore be dynamic.

   (c) [**5 points**]. In order to allow a static universe, Einstein had no choice but to modify his equations for General Relativity, introducing a "fudge factor" known as the "Cosmological Constant." This would be a number, Λ, which is a new constant of nature (on par with, e.g., c, the gravitational constant G, or Planck's constant h) invented solely to allow a static universe. With Λ present, Friedmann's equations become

$$\text{(cosmic acceleration)} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3} \tag{3}$$

$$\text{(expansion rate)}^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \tag{4}$$

Without solving the equations, explain how these new equations with $\Lambda \neq 0$ can allow for a static universe. How is the problem from part (b) overcome?

(d) [**5 points**]. Mathematical details aside, explain how the cosmological must represent a kind of *repulsion* if it can lead to a static universe. For this reason, a nonzero Λ is sometimes referred to as "antigravity."

(e) [**5 bonus points**]. In fact, for the universe to be static, there must not only be a cosmological constant, but also Λ must take a particular value related to the cosmic density. To see this, solve for the value of Λ in a static universe, in which the scale factor is constant and thus always keeps the value is has today: $a = 1$. Is such a universe positively curved, negatively curved, or flat?

(f) [**5 points**]. Consider an Einstein static universe in which the density is now not perfectly homogeneous, so that the actual density $\rho(x, y, z)$ fluctuates from point to point in space around its average value ρ_0. Discuss (but don't calculate) what would happen to a region in which $\rho > \rho_0$ and one in which $\rho < \rho_0$. Comment on the (uncomfortable!) implications for the Einstein static universe.

(g) [**10 points**]. If a cosmological constant exists, it affects not only cosmological goings-on but also any other manifestation of gravity. In particular, if $\Lambda \neq 0$, then in *Newtonian* gravity, the gravitational force between two objects of masses m and M, separated by a distance r, is

$$F_{\text{grav}} = -\frac{GMm}{r^2} + \frac{\Lambda m}{3} r \tag{5}$$

What does it mean physically that the new Λ term has the opposite sign as the usual inverse square term? That is, what do the signs in the force law tell us?

Also, as you consider larger and larger r, which term becomes more important?

Finally, note that fact that the Λ force *increases* with distance $\propto r$, and describe the motion that would result from such a force.

(h) [**5 points**]. Upon Hubble's discovery of galaxy redshifts, Einstein allegedly said that inventing the cosmological constant was his "greatest blunder." How does recent data on cosmic acceleration put this "blunder" in a new context?