Astro 350
Lecture 18
Oct. 10, 2012

Announcements:
- **Discussion 5** due today
- **HW 5** due at start of class Friday
- **Discussion 6** available today, due next Wednesday

Guest Cosmologist today: Prof. Athol Kemball
- key player in future radio telescopes that will make 3-D maps of hydrogen in the Universe
- also does research in General Relativity
- accent way way cooler than B. Fields'

Last time: $E = mc^2$
Today: finish Special Relativity, begin General relativity

Kinetic Energy Generalized

Einstein: energy of particle with mass m and speed v

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

- if $v \neq 0$, then $E > mc^2$:
  speeding up a particle gives it more energy
  extra amount above mc^2 is kinetic energy (Einstein's version)

- if $v \ll c$ but $v \neq 0$, can show (HW 6)

$$\begin{aligned} E &\approx mc^2 + \frac{1}{2}mv^2 && (2) \\ &= \text{(rest energy)} + \text{(Newtonian kinetic energy)} && (3) \end{aligned}$$

- Einstein sez: particles have *same* KE as Newtonian
  if the motion speeds slow compared to c

Q: Why did this conclusion have *to be true?*

In general (haven't proven this, but can):
At slow speeds ($\ll c$):
Special Relativity $\rightarrow$ Galilean/Newtonian physics

Had to be true!

- recall: a theory has to explain *all* data
- Newton was wildly successful – explained all available data until new measurements involving fast speeds (i.e., light)
- new theory (Special Relativity) must explain new data but also must explain all old data!

$\Rightarrow$ so SR must agree with Newton where Newton was successful

The beauty and power of Relativity:

★ *does* give back Newtonian results at slow speeds
"inherits successes of Newton"
★ explains conditions when Newton does and does not work!
★ provides larger, more complete picture!

Cosmology theory will have to respect these ideas of causality!

The Futility of Acceleration to Lightspeed

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

What if high speed, so that $v \rightarrow c$?

- find $E \rightarrow \infty$

Interpretation:

can speed up a mass m, but as speed gets close to c...

- to get particle to go a little faster
  takes more and more energy
- and to get particle to go at c requires *infinite* energy
  physically unachievable, impossible!
- matter (objects with mass) cannot reach speed of light

Q: what if massless particle, $m = 0$*?*

Solve E equation for v:

$$v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \tag{5}$$

if $m = 0$, then $v = c$!

- to move at c, must be massless!
- light particles (photons) are massless!

technical note:

- more general formula works for all particles with mass m and momentum p:

$$E = \sqrt{(mc^2)^2 + (cp)^2} \tag{6}$$

- massive particles have relativistic momentum $p = mv/\sqrt{1 - v^2/c^2}$
- massless particles have $E = cp$
- all particles have $v/c = cp/E \leq 1$

The Cosmic Speed Limit and Causality

all particles (massive or massless) have $v/c \leq 1$
i.e., always have $v \leq c$

speed of light is universal speed limit

particles & information cannot travel faster than c
profound implications for cause & effect ("**causality**")

- an event can only affect future happenings which can be reached by light signal from the event
- this sets "region of future influence" *by* an event
- spacetime regions beyond the reach of a light signal cannot be affected!

Q: what portion of past *spacetime can affect an event?*

An event can only be affected by past events
which could have sent light signals to it
$\rightarrow$ this defines "region of past influence" *on* an event

What's more: events so far apart in space
that they *cannot* be connected with a light signal
are *unaffected* by each other
Q: specifically, what's an event not *affected by your finger snap here and now?*

$\Rightarrow$ key Relativity result/outlook:
- information cannot travel instantaneously
- actions are "local" in the sense that effects transmitted over finite distance in finite time

Extra for the Technorati: Invariants and the Interval

Different observers (typically) disagree about space, time
Is there *anything* they *do* agree about? yes!

Recall: relativistic $\vec{p}$, $E = \sqrt{(mc^2)^2 + (c|\vec{p}|)^2}$: observer-dependent
but: $(mc^2)^2 = E^2 - (c|\vec{p}|)^2$ same for everyone
$\rightarrow$ "invariant" quantity! everyone agrees on its value!

Another key example: two events and their "distance"

- *no* general agreement on separation in time Δt or in space $\Delta \vec{x}$
- *but* everyone agree on the value of $(\Delta s)^2 \equiv (c\Delta t)^2 - |\Delta \vec{x}|^2$, the "interval"

∞ relativity built on relationships among invariant quantities
and how to connect these to experiences of observers
see today's Director's Cut extras for example

Special Relativity Executive Summary

⋆ Special Relativity:
includes high-speed motions (near c), doesn't include gravity

⋆ Space & Time
apparent distances, time intervals, simultaneity
not universal but depend on relative motion

⋆ Energy & Mass
can be converted into each other, mass is form of energy

⋆ Cause & effect ("causality")
- information cannot travel instantaneously
- actions are "local" in the sense that
  effects transmitted over finite distance in finite time

What About Gravity?

Special relativity beautifully accommodates light
(and all of electricity & magnetism)
but ignores gravity

How to include? consider Newton gravity force law

$$F_{\text{grav}} = \frac{GMm}{R^2} \tag{7}$$

gravity force due to mass M depends on present distance R
and spreads over all space ($F \neq 0$ for any $R < \infty$)

Einstein sez: this is totally illegal! an unmitigated disaster!
Q: why? what's the problem?

Newton: mass M exerts force on *any* mass m
determined by *present* distance R

$$F_{\text{grav}} = \frac{GMm}{R^2} \tag{8}$$

but if M moves $\rightarrow$ R changes
Newton's gravity law then implies that
$\rightarrow$ gravity force changes instantaneously over all space!
no signal–including gravity–can move faster than c!

Big Al concludes: *verboten! gotta be wrong!*
major changes needed!

The Equivalence Principle Revisited

How to go about revising gravity? Where to start?

Recall Galileo atop the Tower of Pisa:
gravity $\rightarrow$ all objects move (accelerate) the same way in free fall
regardless of object mass, shape, composition not new result,
but different explanations...

Newton sez:
it just so happens that gravitational mass
the way objects "feel" or "couple to" gravity $F_{\text{grav}} = m_{\text{grav}} g$
is always exactly the same as inertial mass
the way objects resist acceleration $a = F/m_{\text{inert}}$

Einstein sez:
too amazing to be a coincidence, must be deeper...

Einstein's Equivalence Principle

Einstein notes:
Gravity causes acceleration, but in "democratic" way:
all objects accelerate the same

Einstein's Equivalence Principle:
in a closed room, no experiment can distinguish
(non-gravitational) acceleration from gravity
Note similar "feel" to Einstein's Relativity Principle

But note: acceleration is aspect of motion
relates to objects' travel through space and time
$\rightarrow$ gravity=acceleration equivalence will have impact
(i.e., bizarreness) on space and time

Experiments Inside an Accelerating Rocket

Consider a rocket in otherwise empty space

- that is, no gravity!
- moving with constant acceleration a

Experiment:
Astronaut Bart, standing on floor of rocket, has flashlight
holds it at height h, points horizontally, shines towards wall

Sketch experiment

iClicker Poll: Light Beam in Accelerating Rocket

in rocket with constant acceleration

Bart hold flashlight at height h, shoots beam horizontally

At what height will beam hit opposite wall?

A at same h

B higher than h

C lower than h

hint: easier to think about when looking at experiment
from non-accelerating viewpoint

key ideas:
light takes time to move across spaceship
during which, spaceship accelerates $\rightarrow$ gains v, moves vertically

in non-accelerating frame, see that
• light path is straight (horizontal) line
• spaceship vertical motion $\rightarrow$ far wall moved higher
$\Rightarrow$ light hits below where aimed

in accelerating frame (i.e., according to Bart):
agrees that light hits below where aimed, and concludes
⋆ light ray deflected
⋆ entire light path bent (in fact, a parabola!)

Q: but what does this mean, according to Al's Equiv Principle?

Gravitational Lensing

In accelerating spaceship: light rays bent

But by equivalence principle:
must find same result due to gravity, so:

⋆ gravity bends light rays
⋆ light "falls" too!
⋆ gravitating objects "attract" light rays
  distort light paths differently depending on
  how strong the gravity over each path

gravitating objects distort passing light
leads to distorted images of objects behind gravity sources

gravitational lensing

- observable effect, and in fact
- an increasingly powerful tool!

Accelerating Rockets & Clocks

consider "light clocks" installed in spaceship

- manufactured identically in Switzerland
- each emits light pulse every Δt microseconds

clocks, astronauts stationed in ceiling (Milhouse) and floor (Bart) (height difference Δh)

Q: if rocket not *accelerating, do M & B see the other's clock tick at same rate as his own?*

Now fire rockets $\rightarrow$ spaceship has constant acceleration a

Compared to non-acceleration light travel time

Q: does the downgoing flash take longer/shorter/same time?

Q: does the upgoing flash take longer/shorter/same time?

Q: and by the equivalence principle...?

Time Warp: Gravitational Time Dilation

Clocks in accelerating spaceship:
Bart (floor observer) accelerating towards downgoing light
  sees it sooner than if $a = 0$
  B sez M's clocks running fast
Milhouse (ceiling) accelerating away from upgoing light ray
  sees it later than if $a = 0$
  M sez B's clocks running slow

But equivalence principle says: gravity must do same thing! So...

⋆ clocks in basement appear to run slower
    than clocks in attic!
  in fact, attic clocks appear faster by amount
  $\Delta t = t_{\text{attic}} - t_{\text{basement}} = g \Delta h^2 / c^3$ a tiny effect unless g huge
⋆ time "warping" but now due to gravity:
  "**gravitational time dilation**"
⋆ gravity influences "flow" of time!

Light Bending: The Sun

In principle: *all* gravitating objects bend light
including you, me, the earth...
In practice: need strong gravity source to create
effect large enough to observe

Einstein (1915) devised first test: the Sun
- Sun's gravity deflects starlight rays *diagram: paths*
- the stronger the gravity along the path
  the bigger the deflection
  ...in fact, bending angle $\alpha = \dfrac{4GM_\odot}{c^2 R_{\text{closest}}}$

$\Rightarrow$ biggest effect for starlight just "grazing" edge of Sun
Q: why is this technically challenging to see?
Q: how to get around the problem?

1919 Eclipse: Give it up for Big Al!

Problem: Sun's glare obscures surrounding starlight
Solution: block glare with eclipse!

1919: total solar eclipse in Southern hemisphere
expedition led by Sir Arthur Eddington
⋆ starlight bent! Woo hoo!
⋆ relativistic gravity confirmed!
⋆ Einstein an instant celebrity
`www: NYTimes headlines`

Now tested many times, and very accurately
- all starlight bending experiments confirm Einstein!

Moreover, once established, grav lensing is a very powerful tool
Q: why would it be useful?

Director's Cut Extras

For the Technorati: More on the Interval

for two nearby events: different observers with different motions

- *disagree* on event separations in time Δt and space Δx
- but *agree* on **interval** Δs
  which each observer calculates from his/her Δt, Δx:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 \tag{9}$$

Example: observer A snaps fingers twice
all while bystander B sees A move at speed v

Interval according to A

all observers perceive self at rest
$\rightarrow$ $(\Delta x)_A = 0$ and $(\Delta t)_A =$ time between snaps, and
calculates interval $(\Delta s)_A = c(\Delta t)_A$
$\rightarrow$ interval is $c\times$ time diff for observer located at both events!

Interval according to B

bystander B sees A moving at speed v

$\rightarrow$ in time interval $(\Delta t)_B$ sees A move dist $(\Delta x)_B = v(\Delta t)_B$

calculates interval

$$(\Delta s)_B = \sqrt{(c\Delta t)_B^2 - (\Delta x)_B^2} = (c\Delta t)_B \sqrt{1 - \frac{(\Delta x)_B^2}{(c\Delta t)_B^2}} = (c\Delta t)_B \sqrt{1 - \frac{v^2}{c^2}} \tag{10}$$

Invariance

But interval is invariant, so $(\Delta s)_A = (\Delta s)_B$ and thus

$$(\Delta t)_{\text{rest}} = (\Delta t)_A = (\Delta t)_B \sqrt{1 - \frac{v^2}{c^2}} = (\Delta t)_{\text{moving}} \sqrt{1 - \frac{v^2}{c^2}} \tag{11}$$

we recover the time dilation formula!