Astro 350
Lecture 20
Oct. 15, 2012

Announcements:

- **HW 6** due at start of class Friday
- **Discussion 6** due Wednesday
- Guest Cosmologist: Prof. Andrea Ghez
  "The Galactic Center: Unveiling the Heart of our Galaxy"
  Tues Oct 17, 7pm, Lincoln Hall Theater

Last time: gravitational lensing and dark matter

The General Theory of Relativity

1915: Einstein publishes General Theory of Relativity
a.k.a. **General Relativity**, a.k.a. GR
landmark intellectual achievement

keeps all key concepts from Special Relativity
- no absolute space, time
- light always moves at c, matter $< c$
- mass is form of energy
- causality: no particles, signals, info travel $> c$

but now fully includes gravity: GR is the modern theory of gravity

Key GR Idea I:
equivalence principle $\rightarrow$ gravity affects all objects the same
$\rightarrow$ *gravity is not a force but a property of space & time!*
but gravity source is matter, so:
GR is theory connecting matter, space, and time!

If gravity isn't a force, what is it?
hint: already saw that gravity "warps" time

Key GR Idea II:
according to GR, gravity is "curvature" of space & time!?!
i.e., *gravity "warps" both space and time*
→ spacetime "curved"

- gravitational redshifting, time dilation, light bending are all manifestations of this
- curved orbits of particles due to gravity in GR are really responses to spacetime curvature!
- note: gravity = geometry! harkens back to Greeks!

GR Slogans (T-Shirt/bumper sticker/tweet/tattoo):

- matter tells spacetime how to curve
- curvature tells matter how to move

these ideas are beautiful and powerful
but also not (for most people) intuitive or trivial

best way to learn is from examples
will focus on two key examples of relativistic spacetimes

- example #2: the Universe
  rest of the course after today
- today: example #1....

Interlude: Gravitational Energy

Returning briefly to Newton:
recall: forces & energy linked

object of mass m, a distance R from mass M has gravitational force $F_{\text{grav}} = GMm/R^2$, and gravitational potential energy

$$PE_{\text{grav}} = -\frac{GMm}{R} \tag{1}$$

Q: where is PE=0,i.e., at what R? what happens when $R = 0$?
Q: how does PE at ground compare to PE 1 meter high?
Q: dropped object to floor: PE? KE?
Q: bouncing ball: what happens to KE, PE, total E?
Q: why called "potential" energy?

Energy in Motion in Gravitational Fields

objects moving only due to gravity
total energy

$$E_{\text{tot}} = \text{kinetic} + \text{potential} = \frac{1}{2}mv^2 - \frac{GMm}{R} \qquad (2)$$

Special case: object at large distance $\to \infty$, at rest $v = 0$
falls to position R

- *Q: What is E_{tot} at first?*
- *Q: how does KE change? PE? E_{tot}?*

Escape Speed

in example: initially, $KE = 0$ and $PE = 0$ so $E_{\text{tot}} = 0$
and E cons $\rightarrow$ $E_{\text{tot}} = 0$ always

- as falls, speeds up $\rightarrow$ KE$\uparrow$ but
  gets closer $\rightarrow$ $R\downarrow$ $\rightarrow$ $PE\downarrow$ (more negative)
- so always true that $0 = \frac{1}{2}mv^2 - GMm/R$
  which means $v^2 = 2GM/R$, or $v = \sqrt{2GM/R}$

$\star$ gives fall speed v at distance R
e.g., $E = 0$ fall from ∞ to Earth surface:
hit with speed $v = 25,000$ mph! `www: terrestrial craters`

- *Q: to launch object from R to ∞, what v needed?*

to launch object from Earth $\rightarrow$ at rest at ∞:
need final state to have $E_{\text{tot}} = 0$,
so once again: always need $E = 0$

$$E = 0 = \frac{1}{2}mv^2 - \frac{GMm}{R} \tag{3}$$

$\rightarrow$ same case!
$\rightarrow$ need launch with $v_{\text{esc}} = \sqrt{2GM/R}$ **escape speed**

Q: what if launch with $v > v_{\text{esc}}$? with $v < v_{\text{esc}}$?

if launch with $v = v_{\text{esc}}$, then
$E_{\text{tot}} = 0$, and $KE = PE$ always
particle escapes, but at rest when far away ("∞")

If launch with $v > v_{\text{esc}}$ then $KE > PE$
$\to$ when far away, $PE \to 0$ but still $KE \neq 0$
$\to$ particle escapes, still moving when far away
$\to$ $E_{\text{tot}} = KE + PE > 0$ $\to$ particle "unbound"

If launch with $v < v_{\text{esc}}$ then $KE < PE$
$\to$ at some distance, slow down to $v = 0 \to KE = 0$
but still $PE < 0$ $\to$ can't go farther
$\to$ turn around, fall back: particle "bound"
$\to$ $E_{\text{tot}} < 0$: not enough energy to escape

Can predict the future: just ask what is v_{launch} vs v_{esc}?
- if $v_{\text{launch}} < v_{\text{esc}}$: object falls back (pop fly)
- if $v_{\text{launch}} \geq v_{\text{esc}}$: object escapes (rocket)

Black Holes

Laplace (1790's):
escape velocity $v_{\rm esc} = \sqrt{2GM/R}$
What if star of mass M has a radius $R < 2GM/c^2$?
then $v_{\rm esc} > c$!
light cannot escape! $\rightarrow$ black hole

Wrong argument (Newtonian gravitation) ...but right answer!

General relativity predicts existence of black holes
and their properties

Black Hole Properties

any object of *any* mass M can (in principle) become a black hole!

size: Schwarzchild radius

$$R_{\text{Sch}} = \frac{2GM}{c^2} \tag{4}$$

radius also provides BH "recipe":

- crush object M smaller than R_{Sch} $\rightarrow$ get BH!
- example: for mass of Sun $R_{\text{Sch}} = 2GM_{\odot}/c^2 = 3.0$ km
  but actual $R_{\odot} = 7 \times 10^6$ km
  $\rightarrow$ the Sun is not a black hole! (whew!)
- for mass of Earth: $R_{\text{Sch}} = 1$ cm!

The Black Hole Horizon

Why call R_{Sch} the BH radius? nothing is there!

True, but: R_{Sch} marks "point of no return"
horizon: surface enclosing the BH
i.e., horizon is surface of sphere w/ radius R_{Sch}

horizon is one-way "membrane"
once inside $r \leq R_{\text{Sch}}$ nothing can escape...even light!
cosmic roach motel!

Hence:
no light escapes $\rightarrow$ black
but nothing else moves as fast $\rightarrow$ nothing else escapes $\rightarrow$ hole

Life Near a Black Hole

Experiment: lower astronaut (Jodie) near $R_{\rm Sch}$
we are at mission control, far away ($r_{\rm us} \gg R_{\rm Sch}$)
communicate w/ light signals

when viewing photons (or clock ticks)
emitted at $r_{\rm em}$, observed at $r_{\rm obs}$
general rule:

$$\frac{\Delta t_{\rm obs}}{\Delta t_{\rm em}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \sqrt{\frac{1 - R_{\rm Sch}/r_{\rm obs}}{1 - R_{\rm Sch}/r_{\rm em}}} \tag{5}$$

What do we see?

obs=us: $r_{\rm obs} \to \infty$; em=Jodie: $r_{\rm em} > R_{\rm Sch}$

- Jodie's watch: $\Delta t_{\rm obs}/\Delta t_{\rm em} = 1/\sqrt{1 - R_{\rm Sch}/r_{\rm em}} > 1$
  $\rightarrow \Delta t_{\rm obs} > \Delta t_{\rm em}$! appears to tick slow! time dilation!
- wavelengths: $\lambda_{\rm obs} > \lambda_{\rm em}$! redshift!

Q: and Jodie?

What do we see?

intuitively: expect inequalities to reverse...and they do

obs=Jodie: $r_{\rm obs} > R_{\rm Sch}$; em=us: $r_{\rm em} \to \infty$:

- our watches: $\Delta t_{\rm obs}/\Delta t_{\rm em} = \sqrt{1 - R_{\rm Sch}/r_{\rm em}} < 1$
  $\to \Delta t_{\rm obs} < \Delta t_{\rm em}$! appears to tick fast!
- wavelengths: $\lambda_{\rm obs} < \lambda_{\rm em}$! blueshift!

When Jodie returns:

then $r_{\rm em} = r_{\rm obs}$

- $\Delta t_{\rm obs} = \Delta t_{\rm em}$: her watch ticks at same rate as ours!
- but the *elapsed time* is shorter on her watch
  and so she is younger than her twin!

Life Inside a Black Hole

once inside R_{Sch}, no getting out
all matter $\rightarrow$ center $\rightarrow$ point (?): "singularity"
i.e., finite mass M in volume $V = 0 \rightarrow$ density $\rho = M/V \rightarrow \infty$!
D'oh! known laws of physics break down

A few remarks:

- we know that all observers travel to center
- don't know what happens once there
- regardless, certain that you die if you go in
- in a way, it's not a relevant question, since can't get info out even if went in (no Nobel Prize!)
- once crushed to $< 10^{-33}$ cm, quantum mechanics important
  i.e., need quantum theory of relativistic gravity!
  ... but there isn't one...yet
- if you have quantum gravity theory, please tell instructor and we'll publish it (your name may even go first!)