Astro 350

Lecture 22

Oct. 19, 2012

Announcements:

- **HW 6** due
- **HW 7** available, due at start of class next Friday
- **Discussion 7** Wednesday

Cosmo-news:

- planet found in αCen star system
- two black holes found in globular star cluster

Last time: finished black holes

Q: What happens to black holes when they eat?

General Relativity: Executive Summary

General Relativity = Einstein's gravity theory

- agrees with all known experiments/observations (so far)
- gravity $\neq$ force, but rather "spacetime curvature"
- matter tells spacetime how to curve
  curvature tells matter how to move
- in GR, the nature of spacetime:
  - ▷ is dynamic, i.e., spacetime responds to matter within it
  - ▷ cannot be deduced from pure thought, but
  - ▷ must be experimentally measured/mapped,
    i.e., the nature of space and time can be different
    depending on the matter/energy content of the Universe;
    have to measure our Universe and its contents
    to see what we have been dealt

Mapping the Universe

Q: how are galaxies arranged in the Universe? Q: what's the cosmological principle? is it true?

Real Galaxies in the Real Universe

Beyond the First Approximation

cosmo principle a very good approximation
on large scales ($\gtrsim$ 50 Mpc)
`www: 2dF`

but do observe fluctuations around average galaxy density
`www: 2dF maps`
on small to medium scales ($\lesssim$ 50 Mpc),
galaxies **clustered** in space:

- loners: “field” galaxy
- few ($\lesssim$ 50) galaxies: group
- 100’s-1000’s of galaxies: cluster
- assemblies of groups and clusters: supercluster

The Logic of the Cosmo Principle

Cosmo Principle:
On large scales ($\gtrsim 50$ Mpc), universe is

- homogeneous $\rightarrow$ smooth
- isotropic

Q: do you need both?
Q: e.g., how can you be isotropic but not homogeneous?
Q: e.g., how can you be homogeneous but not isotropic?

Cosmo principle as cosmic democracy:
Universe has no center, no edge
no special places, directions!

The cosmo principle, in song

I’m just average, common too
I’m just like him, the same as you
I’m everybody’s brother and son
I ain’t different from anyone
It ain’t no use a-talking to me
It’s just the same as talking to you.

Cosmologist Prof. Bob Dylan (1964)

Cosmological Principle: Implications

- demands enormous regularity
"maximal symmetry" $\rightarrow$ simplifies analysis!

- places stringent constraints on
(i.e., simplifies!) the possible nature and behavior
of the Universe and its contents
i.e., is "the cosmologist's friend"

- "trying to tell us something"
about how universe formed?
(e.g., cosmic inflation in early universe?)

iClicker Poll: Cosmodynamics

galaxies have mass $\rightarrow$ gravitate
in general, expect galaxies to be in motion

What pattern of motions (relative to us) will we find?

A most galaxies move towards us

B roughly half move away, half towards us

C most galaxies move away from us

iClicker Poll: Cosmodynamics Twofer

in fact: the *majority* of galaxies move away from us

What percentage of galaxies are observed to move away?

A between 50% and 75%

B between 75% and 90%

C between 90% and 95%

D between 95% and 99%

E $>$ 99%

Cosmodynamics I

of > 1 million galaxies with redshift/blueshift measurements

< 20 galaxies have blueshifts! (only nearest ones)

so: $> 99.9999\%$ of galaxies have redshifts!

$\Rightarrow$ essentially all galaxies have **redshift**:

$$z \equiv \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{rest}}}{\lambda_{\mathrm{rest}}} > 0 \tag{1}$$

$\rightarrow$ move away!

line-of-sight speed: Doppler law sez $v = cz$

first approximation:

Hubble (1929) v & distance r related:

`www: Hubble original data`

Q: how are v and r related mathematically?

Hubble: galaxy speed and distance *proportional*

$\Rightarrow v \propto r$

$\boxed{v = H_0 r}$ Hubble law

in fact: $\vec{v} = H_0 \vec{r}$

that is, speed and distance directions the same

$\rightarrow$ galaxies all move *radially* away from us!

Q: why did it have to be this way?

Hubble Law $v = H_0 r$

Hubble parameter (a.k.a. "Hubble constant")

$$H_0 \simeq 72 \text{ km s}^{-1} \text{ Mpc}^{-1} \tag{2}$$

e.g., galaxy at $r = 10$ Mpc moves away at 720 km/s

Try it!
draw field with MW, other galaxies, $\vec{v}$
Comment on pattern

Note: to zeroth order, z + Hubble law $\to r$
distance measure

Structure + Dynamics: Evolution

observe:

- U. homogeneous, isotropic
- Hubble law $v = Hr$

i.e., galaxies smoothly spread in space, yet moving too
and motions are all directed away from us!
i.e., galaxy velocity pattern “points back to us”

Q: how reconcile?
at least 2 logical possibilities...

1. "Egoist" interpretation: we are at the center of U.
Imagine galaxies all launched from same point (here)
initially: each launched with different speed v_{gal}
afterwards: each coasts, keeping its $v_{\mathrm{gal}} = const$

Then after time t, a galaxy seen at distance $r = v_{\mathrm{gal}} t$
so $r \propto v_{\mathrm{gal}} \Rightarrow$ farther = faster: Hubble!

In this picture: Hubble law means $r = v_{\mathrm{gal}} t = H_0 r t$
so "coasting time " is $t_{\mathrm{H}} = 1/H_0 = 14 \times 10^9$ yr = 14 billion yrs
"Hubble time"* – "egoist" age of Universe

and since max "launch" speed is $v_{\mathrm{gal}} < c$
expect "edge" of galaxy sphere
at radius $d_{\mathrm{H}} = c t_{\mathrm{H}} = c/H_0 = 4200$ Mpc
"Hubble Radius/Length"* – "egoist" size of Universe

*When egoism is discarded, we'll reinterpret the Hubble length & time, but still find both useful & interesting numbers

So “egoist” picture gives Hubble’s law!

Logically possible! But...

Q: give a philosophical reason why we don’t believe this

Q: give a physical reason why this treatment can’t be right?

Q: give an observational reason why we don’t believe this

Critiques of Cosmic Egoism

We are at the center of the universe?

Philosophically:

• not Copernican (violates "principle of mediocrity")

Physically:

• haven't included gravity!

Observationally:

• Milky Way, local galaxies don't look special

not what expect from center of explosion

compare supernova $\rightarrow$ distinctive neutron star/BH at center

• no evidence for "edge" to Universe at great distances

The Magic of Hubble

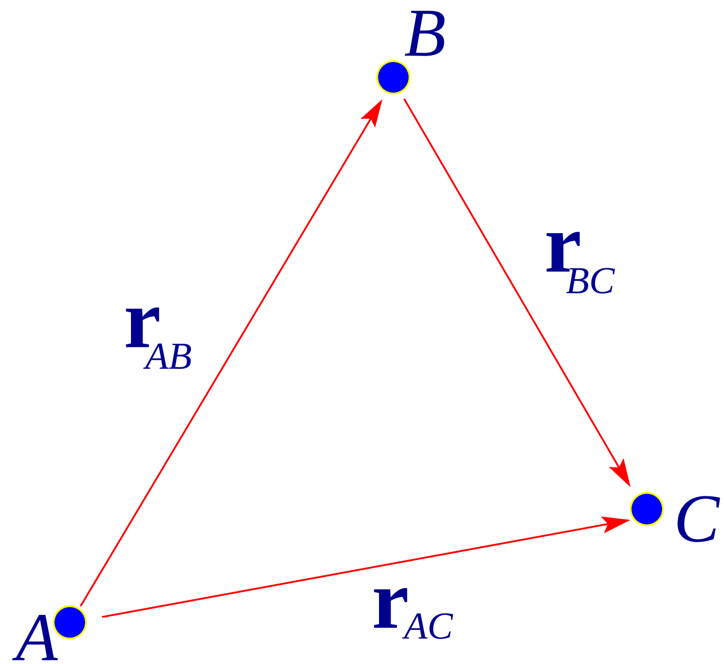
Slightly technical derivation:

consider three arbitrary cosmic points:

$\vec{r}_{BC} = \vec{r}_{AC} - \vec{r}_{AB}$

Assume A sees Hubble's law:

- $\vec{v}_{AB} = H\vec{r}_{AB}$
- $\vec{v}_{AC} = H\vec{r}_{AC}$



Then ask: *what does B see? C?*

find velocities relative to B:

$\vec{v}_{BC} = \vec{v}_{AC} - \vec{v}_{AB} = H(\vec{r}_{AC} - \vec{r}_{AB}) = H\vec{r}_{BC}$

This is huge!

Q: why? What have we proven?

we have shown:

if A sees Hubble's law, then so do (arbitrary) B and C

thus: if *any* observer measures Hubble's law
then *all* observers will measure Hubble's law!

so: Hubble law implies

$\rightarrow$ *all* galaxies recede according to same law

$\rightarrow$ no need for center, space has no special points

Moreover: Hubble law is *only* motion
which preserves homogeneity and isotropy

i.e., *any* other motion breaks cosmo principle

...but Hubble law is exactly what's observed!

Coincidence? I think not! $\rightarrow$ trying to tell us something!

Leads to the other interpretation of Hubble's Law...

2. Einstein interpretation of Hubble's law:

using General Relativity:

$\boxed{\text{Universe is expanding}}$

that is, **space itself is expanding**!

recall: this is possible, since GR says spacetime is dynamic!

But this implies that

- all galaxies receding from all others
- and they do so because they are "riding" on points within an expanding grid!
  imagine rubber graph paper being stretched!

bold, strange idea!

transparency demo: photocopy universe

Q: implications?

Expansion and Cosmology

All of cosmology is nothing more or less
than the evolution of a system that is

- homogeneous
- isotropic
- expanding

⋆ much of cosmology amounts to imagining a box

- filled homogeneously with galaxies (today)
  or atoms/particles (in the early Universe)
- with other identical expanding boxes on all sides

and asking: *how do the contents respond as the box expands?*

⋆ to do this don't need to know if U. has finite or infinite volume!
question is interesting but can distract and confuse

Describing Expansion

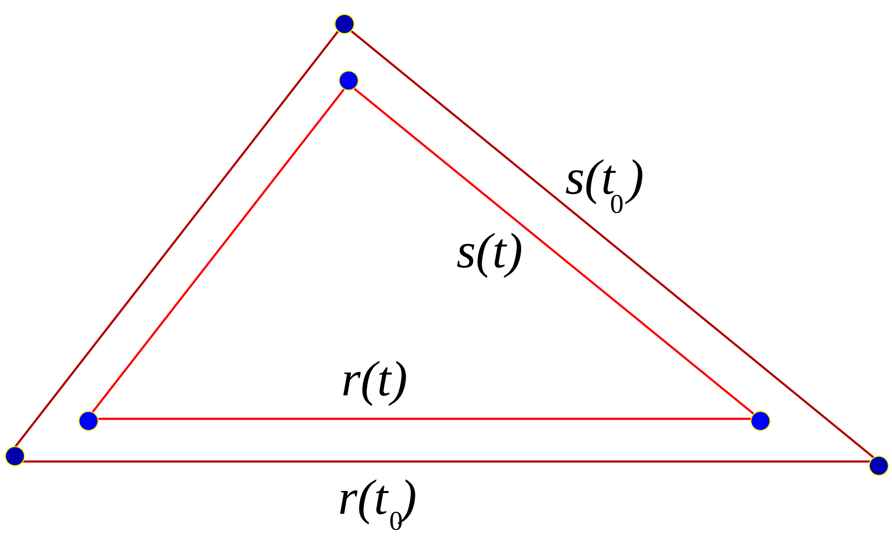
consider triangle defined by 3 observers at t_1

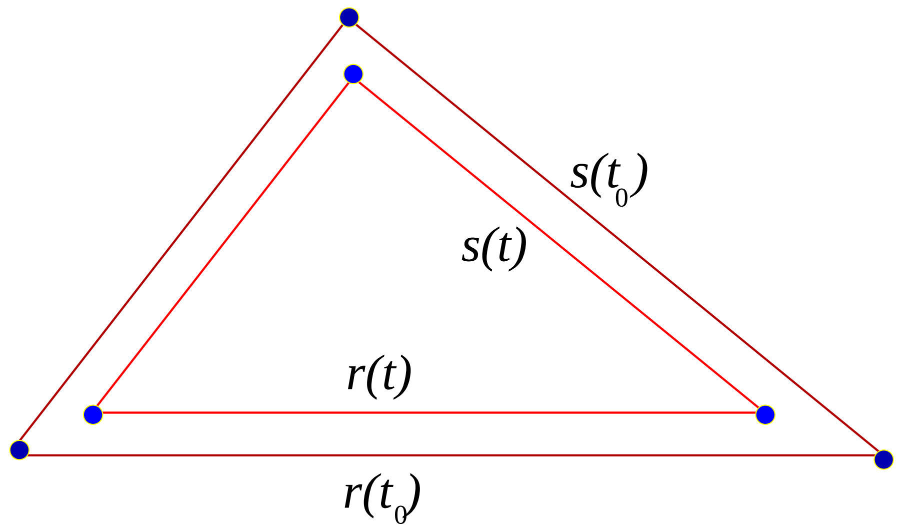
if homogeneous and isotropic expansion

at any later time t_2, new triangle *must always be*

"similar to" original triangle

i.e., have same "shape" – same angles, ratios of sides *Q: why?*





similar $\rightarrow$ ratio C/B always the same so

$$\frac{r(t_1)}{s(t_1)} = \frac{r(t_2)}{s(t_2)} \tag{3}$$

rearrange:

$$\frac{r(t_2)}{r(t_1)} = \frac{s(t_2)}{s(t_1)} \tag{4}$$

Q: what does this imply? Hint: must work for any *triangle!*

cosmo principle $\rightarrow$ triangle *must be* similar:
if stretched more in one direction $\rightarrow$ expansion not isotropic
there would be a preferred direction

since for *any* triangle at *any* two times t_1, t_2

$$\frac{r(t_2)}{r(t_1)} = \frac{s(t_2)}{s(t_1)} \tag{5}$$

then these ratios must have a universal (triangle-indep) value!
and *any* length ℓ changes with time so that

$$\frac{\ell(t_2)}{\ell(t_1)} = \frac{a(t_2)}{a(t_1)} \tag{6}$$

where $a(t)$ must be universal **scale factor**
measures stretching of space due to expansion

The Cosmic Scale Factor

Define:

- $\boxed{t_0 = \textbf{present age of Universe} = \text{“time on cosmic clock”}}$
- ℓ_0=**present distance** between any two objects A & B
  obviously, value is different depending on
  which objects you wish to describe!
- $a(t_0) = 1$: can set *present* scale factor to 1

then at *any time* t, distance ℓ between A and B is

$$\ell(t) = \quad a(t) \quad \times \quad \vec{\ell}_0$$

AB distance at t	scale factor	present AB distance
time varying	*time varying*	*fixed once and for all*

note: haven't said yet how a changes with t

but: if U. expanding *Q: what does this mean for* $a(t)$*?*

What can you say about $a(t_0/2)$*?* $a(2t_0)$*?*

since all lengths change as $\ell(t) = a(t)\ell_0$
expanding U $\rightarrow \ell(t)$ increases with t
$\rightarrow$ scale factor $a(t)$ increases with t
and since we have $a(t_0) = 1$, then
  $a(t_0/2) < 1$ i.e., $a < 1$ in the past, and
  $a(2t_0) > 1$ i.e., $a > 1$ in the future

So: entire history of the universe
contained in the details of
how scale factor a grows with time!
e.g., at some time in past $a = 1/2$: "galaxies twice as close"
  but haven't yet worked out *when* $a(t) = 1/2$!
  we'll get to that directly...

Q: how do cosmic volumes depend on a?
e.g., *Q: when $a = 1/2$?*

Expansion and Areas, Volumes

consider a cube, galaxies at corners
present side length L_0
diagram: cube, label L_0, expansion arrows
$\rightarrow$ cube is "comoving" w/ expansion
draw arrows
volume $V \propto a^3$
$\rightarrow V = L^3 = L_0^3 a^3 = V_0 a^3$
side area $A = A_0 a^2$

So: back when $a = 1/2$, then $V = \frac{1}{8} V_0$, $A = \frac{1}{4} A_0$
galaxies compressed to volume 8× smaller!

`www: raisin cake analogy`
`www: balloon analogy`
Q: what is tricky, imperfect about each analogy?

Expansion: Einstein $\rightarrow$ Hubble

Somewhat technical derivation:
for two arbitrary observers (e.g., "galaxies")
scale factor gives distances $\vec{r}(t) = \vec{r}_0 a(t)$
so velocity is

$$\vec{v}(t) = \Delta\vec{r}/\Delta t = d\vec{r}/dt \equiv \dot{\vec{r}} = \vec{r}_0 \dot{a} \tag{7}$$

with shorthand notation: time rate of change $\dot{a} = da/dt$

but we can rewrite this as

$$\vec{v}(t) = \frac{\dot{a}}{a}\, a\vec{r}_0 = H(t)\vec{r} \tag{8}$$

Q: which means?

We have proven that at time t

observers at distance r recede at speed

$$v(t) = \frac{\dot{a}}{a} \, a \, r_0 = H(t) \, r(t) \tag{9}$$

which means...

$\Rightarrow$ In expanding U, everyone observes Hubble law!

- now interpret "Hubble parameter" $H(t)$ as

$$\boxed{H(t) = \frac{\dot{a}}{a} = \frac{\text{change in cosmic size}}{\text{cosmic size}}} \tag{10}$$

  expansion rate at time t
- $H(t_0) = H_0 =$ expansion rate today
- but expansion rate need not be (and usually isn't) constant!