Astro 350

Lecture 23

Oct. 22, 2012

Announcements:

- **HW 7** due at start of class Friday
- **Discussion 7** due Wednesday

Guest Cosmologist today: Dr. Brett Hayes
expert on observational cosmology and large-scale structure

Last time: Hubble's law
Q: What's Hubble's law? What does it say in simple terms?
What's the pattern of galaxy motions relative to us?
What are possible interpretations of this motion?

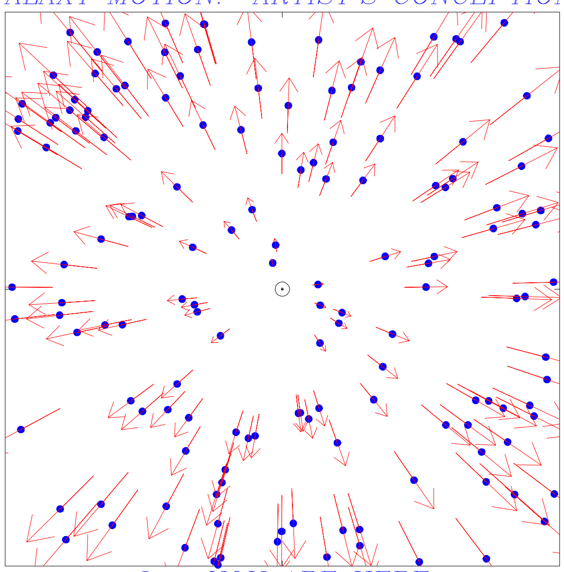
Edwin Hubble (1929):

- (essentially) *all* galaxy spectra show *redshifts*
  $\Rightarrow$ all galaxies move *away* from us
- moreover: *farther galaxies move faster*
- mathematically: velocity v and distance r related by

  $$v = H_0 \, r$$

  with Hubble "constant" $H_0 = 72 \text{ km sec}^{-1} \text{ Mpc}^{-1}$

GALAXY MOTION: ARTIST'S CONCEPTION



⊙ = YOU ARE HERE

Galaxies in the universe, and thus the Universe itself displays a highly ordered motion!

This cries out for interpretation!

1. **"Egoist" interpretation of Hubble's law** *We are at the center of the Universe*

Hubble "flow" can be explaied
if all galaxies exploded outward from here

- if so: age of Universe = "Hubble time" $1/H_0 = 14$ billion yrs
  ...which turns out to be nearly correct!

- but: this is an anti-Copernican outlook

- and no evidence we were the center of an explosion

2. Einstein interpretation of Hubble's law:

using General Relativity:

Universe is expanding

that is, **space itself is expanding**!

recall: this is possible, since GR says spacetime is dynamic!

But this implies that

- all galaxies receding from all others
- and they do so because they are "riding" on points within an expanding grid!
  imagine rubber graph paper being stretched!

bold, strange idea!

demo: expanding universe

Q: implications?

The Magic of Hubble

Somewhat technical derivation:

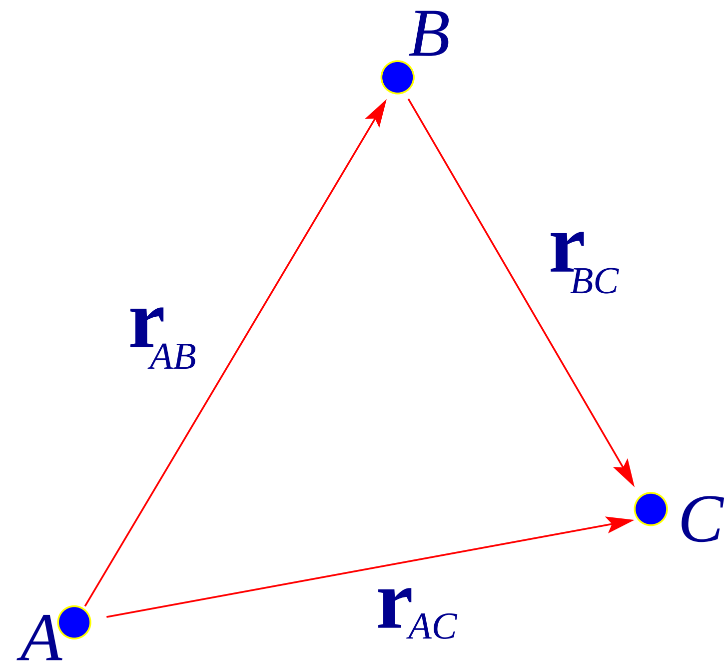
consider three arbitrary cosmic points:

$\vec{r}_{BC} = \vec{r}_{AC} - \vec{r}_{AB}$

Assume A sees Hubble's law:

- $\vec{v}_{AB} = H\vec{r}_{AB}$
- $\vec{v}_{AC} = H\vec{r}_{AC}$

Then ask: *what does B see? C?*



find velocities relative to B:

$\vec{v}_{BC} = \vec{v}_{AC} - \vec{v}_{AB} = H(\vec{r}_{AC} - \vec{r}_{AB}) = H\vec{r}_{BC}$

This is huge!

Q: why? What have we proven?

we have shown:

if A sees Hubble's law, then so do (arbitrary) B and C

thus: if *any* observer measures Hubble's law

then *all* observers will measure Hubble's law!

so: Hubble law implies

$\longrightarrow$ *all* galaxies recede according to same law

$\longrightarrow$ no need for center, space has no special points

Moreover: Hubble law is *only* motion

which preserves homogeneity and isotropy

i.e., *any* other motion breaks cosmo principle

...but Hubble law is exactly what's observed!

Revolution Re-Re-Re-Visited

Copernican Revolution I (17th Century):
Earth is one typical planet among many
not center of solar system

Copernican Revolution II (earth 20th Century):
Sun is one typical star among many
not center of Milky Way Galaxy

Copernican Revolution III (1920's):
Milky Way is one typical galaxy among many
Universe much larger than previously thought

Copernican Revolution III (late 20th century):
most matter in the U is weakly interacting dark matter
we are not even made of the dominant stuff

Copernican Revolution IV (20th century):
Universe is homogeneous on large scales,
and has no center

... stay tuned for more...

Expansion and Cosmology

All of cosmology is nothing more or less
than the evolution of a system that is

- homogeneous
- isotropic
- expanding

⋆ much of cosmology amounts to imagining a box

- filled homogeneously with galaxies (today)
  or atoms/particles (in the early Universe)
- with other identical expanding boxes on all sides

and asking: *how do the contents respond as the box expands?*

∞ ⋆ to do this don't need to know if U. has finite or infinite volume!
question is interesting but can distract and confuse

Describing Expansion

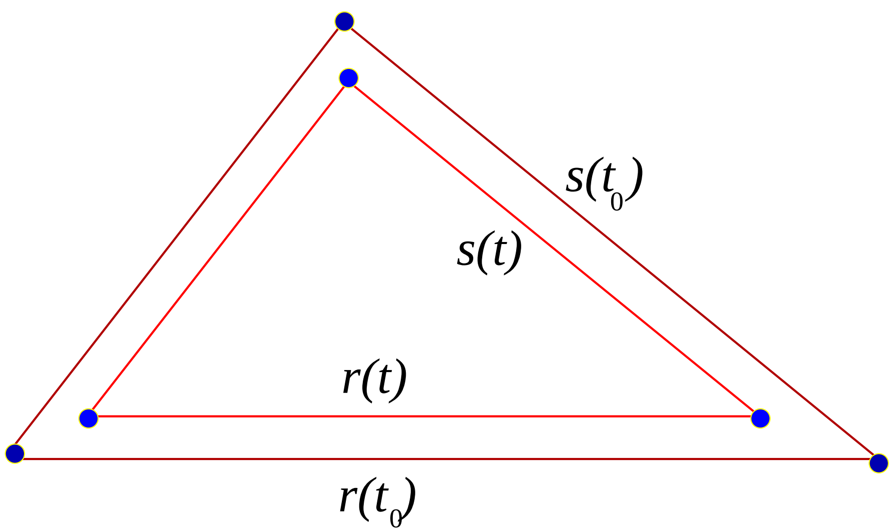
consider triangle defined by 3 observers at t_1

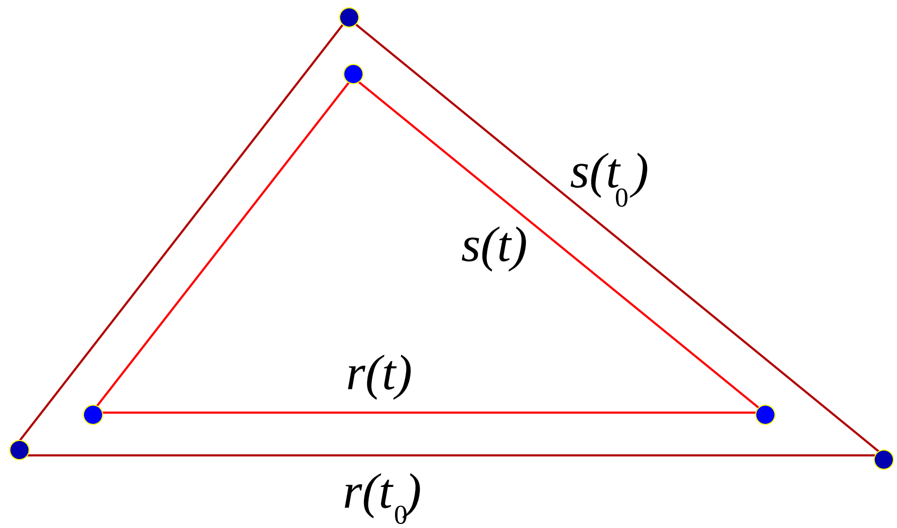
if homogeneous and isotropic expansion

at any later time t_2, new triangle *must always be*

"similar to" original triangle

i.e., have same "shape" – same angles, ratios of sides *Q: why?*





similar $\rightarrow$ ratio C/B always the same so

$$\frac{r(t_1)}{s(t_1)} = \frac{r(t_2)}{s(t_2)} \quad (1)$$

rearrange:

$$\frac{r(t_2)}{r(t_1)} = \frac{s(t_2)}{s(t_1)} \quad (2)$$

Q: what does this imply? Hint: must work for any *triangle!*

cosmo principle $\to$ triangle *must be* similar:
if stretched more in one direction $\to$ expansion not isotropic
there would be a preferred direction

since for *any* triangle at *any* two times t_1, t_2

$$\frac{r(t_2)}{r(t_1)} = \frac{s(t_2)}{s(t_1)} \tag{3}$$

then these ratios must have a universal (triangle-indep) value!
and *any* length ℓ changes with time so that

$$\frac{\ell(t_2)}{\ell(t_1)} = \frac{a(t_2)}{a(t_1)} \tag{4}$$

where $a(t)$ must be universal **scale factor**
measures stretching of space due to expansion

A Gut Feeling for the Cosmic Stretch

consider two concentric circles
the smaller with radius $r_{\text{today}} = 1$ Mpc today
and the larger with radius $R_{\text{today}} = 2$ Mpc today

at some past time, the larger circle had radius $R_{\text{past}} = 1\text{Mpc} = 12R_{\text{today}}$
Q: at that time, what was the radius of the smaller *circle?*
at some future time, the larger circle will have $R_{\text{future}} = 4\text{Mpc} = 2\text{R}_{\text{today}}$
Q: at that time, will will be radius of the smaller *circle?*

Q: what does this tell us about how radii change with time?

this rule has to apply for radii in all directions *Q: why?*
Q: what does this tell us about how distances *change with time?*

In the concentric circle example:
large circle must keep radius $R = 2r$ at *all times*
or else expansion would not keep the Universe homogeneous

this means that that when the larger circle had radius
$R_{\text{past}} = 1\text{Mpc} = 12 R_{\text{today}}$
it must be that the smaller circle had
$r_{\text{past}} = 12 r_{\text{today}} = 0.5\ \text{Mpc}$

similarly, when the large circle will have radius
$R_{\text{future}} = 4\text{Mpc} = 2 R_{\text{today}}$
it must be that the smaller circle will have
$r_{\text{future}} = 2 r_{\text{today}} = 2\ \text{Mpc}$

And thus in general, *all* circle radii
are shrunken or enlarged by one, same, universal factor

$$r(t) = \text{universal stretch factor} \times r_{\text{today}} \tag{5}$$

moreover: this should apply to *all* cosmic distances!

The Cosmic Scale Factor

Define:

- t_0=**present age of Universe**= "time on cosmic clock"
- ℓ_0=**distance** between any two objects A & B
  as measured **today**, at the present time
  obviously, value is different depending on
  which objects you wish to describe!

then at *any time* t, distance ℓ between A and B is

$$\ell(t) = \quad a(t) \quad \times \quad \vec{\ell}_0$$

AB distance at t	scale factor	present AB distance
time varying	*time varying*	*fixed once and for all*

note: haven't said yet how a changes with t

but: if U. expanding *Q: what does this mean for $a(t)$?*

What can you say about $a(t_0/2)$? $a(2t_0)$?

cosmic scale factor $a(t)$ measures how distances or lengths change with time
$\rightarrow$ cosmic "stretch-o-meter" or "copier enlargement setting"

since all lengths change as $\ell(t) = a(t)\ell_0$
expanding U $\rightarrow$ $\ell(t)$ increases with t
$\rightarrow$ scale factor $a(t)$ increases with t
and since we have $a(t_0) = 1$, then
$a(t_0/2) < 1$ i.e., $a < 1$ in the past, and
$a(2t_0) > 1$ i.e., $a > 1$ in the future

So: entire history of the universe contained in the details of how scale factor a grows with time!
e.g., at some time in past $a = 1/2$: "galaxies twice as close"
but haven't yet worked out *when* $a(t) = 1/2$!
we'll get to that directly...

Q: how do cosmic volumes depend on a?
e.g., *Q: when $a = 1/2$?*

Expansion and Areas, Volumes

consider a cube, galaxies at corners
present side length L_0
diagram: cube, label side L_0
$\rightarrow$ cube is "comoving" w/ expansion
expansion arrows
volume $V \propto a^3$
$\rightarrow V = L^3 = L_0^3 a^3 = V_0 a^3$
side area $A = A_0 a^2$

So: back when $a = 1/2$, then $V = \frac{1}{8} V_0$, $A = \frac{1}{4} A_0$
galaxies compressed to volume 8× smaller!

`www: raisin cake analogy`
`www: balloon analogy`
Q: what is tricky, imperfect about each analogy?

Expansion: Einstein $\rightarrow$ Hubble

Somewhat technical derivation:

for two arbitrary observers (e.g., "galaxies")

scale factor gives distances $\vec{r}(t) = \vec{r}_0 a(t)$

so velocity is

$$\vec{v}(t) = \Delta\vec{r}/\Delta t = d\vec{r}/dt \equiv \dot{\vec{r}} = \vec{r}_0 \dot{a} \tag{6}$$

with shorthand notation: time rate of change $\dot{a} = da/dt$

but we can rewrite this as

$$\vec{v}(t) = \frac{\dot{a}}{a}\, a\vec{r}_0 = H(t)\vec{r} \tag{7}$$

Q: which means?

We have proven that at time t
observers at distance r recede at speed

$$v(t) = \frac{\dot{a}}{a}\ a\ r_0 = H(t)\ r(t) \tag{8}$$

which means...

⇒ In expanding U, everyone observes Hubble law!

- now interpret "Hubble parameter" $H(t)$ as

$$\boxed{H(t) = \dot{a}/a} \tag{9}$$

expansion rate at time t

- $H(t_0) = H_0 =$ expansion rate today
- but expansion rate need not be (and usually isn't) constant!

Redshifts

wavelengths are *lengths*!

...it's right there in the name!

expansion stretches photon λ

$\lambda \propto a$

if emit at $t_{\rm em}$, then

$\lambda(t) = \lambda_{\rm emit} a(t)/a(t_{\rm em})$

if observe later,

$\lambda_{\rm obs} = \lambda_{\rm em}\ a_{\rm obs}/a_{\rm em}$

measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1 - a_{\rm em}}{a_{\rm em}} \quad \Rightarrow \quad a_{\rm em} = \frac{1}{1+z}$$

Scale factor $\leftrightarrow$ redshift

$a = 1/(1+z)$

$z = 1/a - 1$

`www: Sloan Digital Sky Survey spectra`

`www: quasar recordholder`

Example: most distant quasar has $z = 6.4$
$\rightarrow$ scale factor $a = 1/(1+6.4) = 0.135$
interparticle (intergalactic) distances 13.5% of today!
$\rightarrow$ galaxies 1+6.4=7.4 times closer
squeezed into volumes $(7.4)^3 = 400$ times smaller!

Recall from General Relativity, black hole discussions
gravitational redshifting often accompanied by...
Q: what? and how might you observe this?

Cosmic Time Dilation

GR: gravitational redshifting goes hand-in-hand with gravitational time dilation
$\rightarrow$ i.e., redshifted objects also appear to have slow clocks
and blueshifted objects appear to have fast clocks

Cosmic time dilation observed! And only recently!
Challenge: need "standard clock" in order to know
that it's running slow
Tool: exploding stars (supernovae) – know timing of brightness
observe high-z supernovae, see lengthening of
duration in explosion and aftermath!
Woo hoo!

Q: how does expansion affect photon energy?
Q: for blackbody, how does expansion affect T?
hint: $T \leftrightarrow \lambda$ connection?

Expansion and Radiation Energy & Temperature

since $E_\gamma = hc/\lambda \propto 1/\lambda$, then
$E_\gamma \propto 1/a \rightarrow$ photon energy redshifts, i.e., decreases with time

for thermal radiation, Wien's law: $T \propto 1/\lambda_{\text{max}}$ so $T \propto 1/a \Rightarrow T$ decreases $\rightarrow$ U cools!
the universe cools as it expands

today: cosmic thermal radiation peaks at $\lambda \sim 1$ mm
"cosmic microwave background" radiation (CMB)
CMB temperature today: $T_0 = 2.725 \pm 0.001$ K
≈ 3 degrees above absolute zero

in past $\rightarrow$ CMB, universe hotter:
distant but still "garden variety" quasar: $z = 3$
"feels" $T = 8$ K (effect observed!)