Astro 350

Lecture 26

Oct. 29, 2012

Announcements:

- **HW 7** due now (extended!)
- good news: no HW or discussion this week
- bad news: Hour Exam 2 on Friday, info online
- apologies for canceled class Friday

Last times:

Q: what is the cosmic scale factor?

Q: what determines how it changes with time?

Q: how is the Universe like a pop fly?

Q: time changes of galaxy speeds in empty U?

Q: time changes of galaxy speed in U with matter?

Q: fate of U with low matter density ρ? high ρ?

at *any time* t, distance ℓ between A and B is

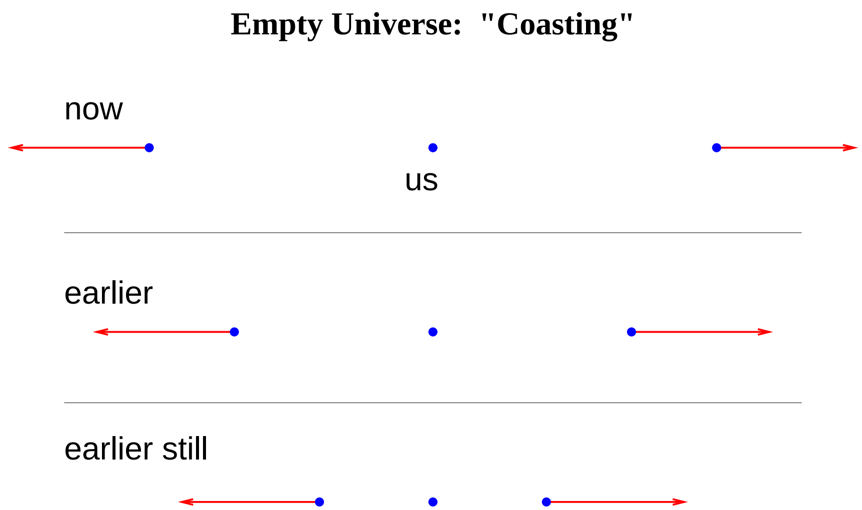
$$\ell(t) = \quad a(t) \quad \times \quad \vec{\ell}_0$$

AB distance at t	scale factor	present AB distance
time varying	*time varying*	*fixed once and for all*

cosmic time *today* $t = t_0$: scale factor $a(t_0) = 1$
in cosmic *past* $t < t_0$: scale factor $a(t) < 1$
in cosmic *future* $t > t_0$: scale factor $a(t) > 1$

Matter-Free "Empty" Universe

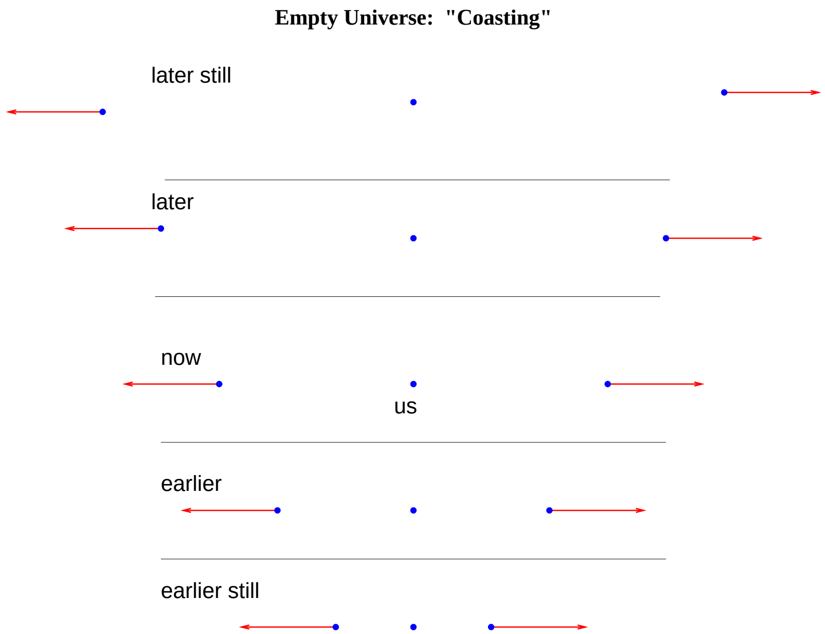
No matter → no attraction between galaxies
→ nothing to change galaxy speeds
→ galaxies "coast" keeping constant velocity



expansion rate:
neither accelerated **nor** decelerated

Q: plot of a vs t for this universe?
Q: fate of such a universe? Why?

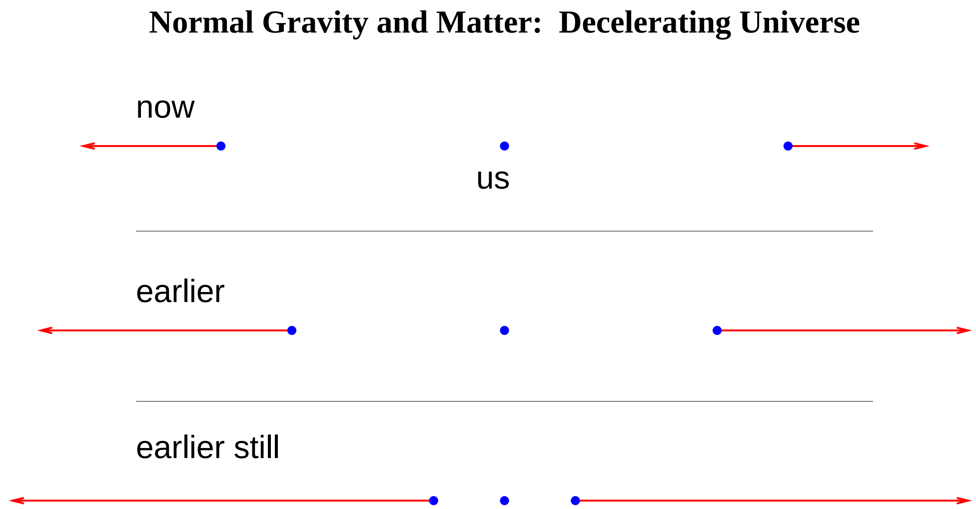
empty universe fate:
no matter $\rightarrow$ no gravity to slow galaxy motions
the **universe expands forever**!



everything ever more spread out
Universe becomes ever more empty and cold
$\rightarrow$ known as "**the Big Chill**"

Matter-full Universe

The real universe has galaxies with mass $\rightarrow$ attract each other
$\rightarrow$ inward gravity slows expansion
$\rightarrow$ speeds constantly *decreasing*, galaxies *decelerating*
$\Rightarrow$ to achieve observed speed today, had to be *faster* in past!

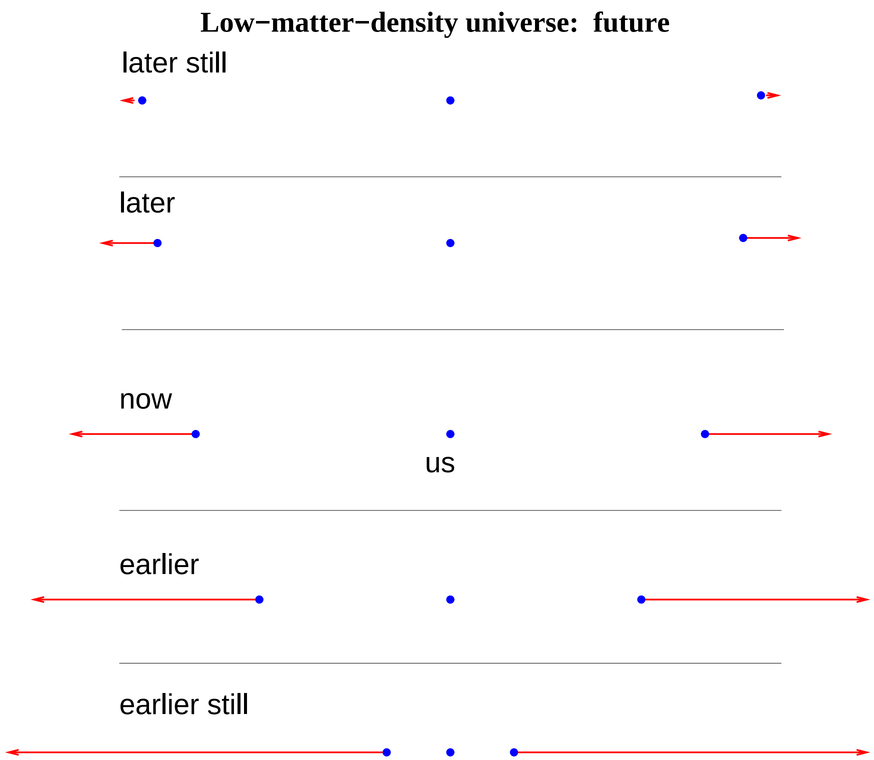


expansion rate: **decelerated**
Q: plot of a vs t for this universe?
Q: fate of such a universe? why?

Low-Density Universe: Fate

in a *low*-density universe: weak gravity

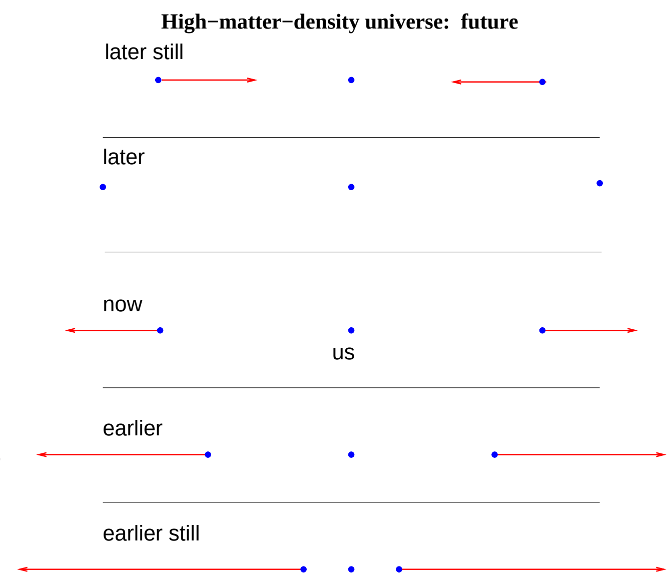
- expansion slows, but never stops
- fate: expand forever, but speeds slower than now



High-Density Universe: Fate

in a *high*-density universe:

- high density $\rightarrow$ strong gravity
  strong enough to overcome inertia!
- expansion slows until *stopping* momentarily
- but gravity will not stop!
  galaxies still attract each other!
- galaxies now move *toward* each other
- Universe begins to *contract*
  as they get closer, gravity stronger $\rightarrow$ galaxies faster
- continues until Universe *collapses* on itself!
  fate known as the **Big Crunch**



Q: plot of a vs t for this universe?

Q: what lessons do we draw about cosmic history and evolution?

Density and Destiny

We have seen:

- a high-density universe has a different expansion history than a low-density universe
- namely: a normal-matter high-ρ universe decelerates & slows more rapidly than a low-ρ universe and expanded *even faster* in the past
- the future fate of the cosmos is very different depending on the cosmic density

Lessons:

- different cosmic fates are possible!
- the evolution and fate of the Universe depends on what's in the Universe
- namely: cosmic fate depends on cosmic density
- weight is fate! density is destiny!

The Friedmann Equations

Friedmann Acceleration Equation

$$\text{cosmic acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \qquad (1)$$

important features:

- *Q: significance of $-$ sign?*

Friedmann Equation (“Energy Eq.”)

$$(\text{cosmic expansion rate})^2 = H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \qquad (2)$$

where K is a constant

- *Q: how does expansion rate depend on contents of U?*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{3}$$

note $-$ sign:

- due to attractive nature of gravity
- galaxy gravity on each other slows expansion

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \tag{4}$$

- for any time t, relates expansion rate $H(t)$ = change in a to constant K and values of $\rho(t), a(t)$ at t
- cosmic contents (density) influences expansion
- K term can be important – or zero!
  value, sign of constant K has to be measured

Q: so what's the big picture–what just happened?

Post-Math Aftermath

What just happened?

Inputs:

- Newton's laws
- homogeneous, isotropic Universe, that is
- expanding

Outputs: Friedmann equations

expressions for how scale factor a changes with time

- expansion rate: time change of a
- acceleration rate: time change of expansion

These give a precise mathematical statement
of what we already found:
in the expanding universe: galaxies have inertia
resist change in speed gravity acts to change speeds

Here endeth material on Hour Exam 2

Studying the History of Cosmic Expansion

Recall: Friedmann sez that at any cosmic "moment"
("epoch"− measured by time t and/or redshift z)
the cosmic expansion H directly related to
cosmic contents ρ

So: imagine we could measure expansion rate H
at many different epochs, finding $H(z)$ at many z

this determines the cosmic expansion history
then Friedmann $\rightarrow$ density history, e.g, $\rho(z)$

$\star$ cosmic motion $\leftrightarrow$ cosmic contents
$\star$ expansion measures cosmic density!
$\star$ can compare with expectations if $\rho = \rho_{\text{matter}} + \rho_{\text{radiation}}$

iClicker Poll: Effect of Dark Matter

We have seen that galaxies are *mostly* made
of *dark matter*, which holds galaxies together

compared to a universe with only the visible galaxies
the effect of dark matter should be to?

A add gravity, slowing expansion over time

B add gravity, speeding up expansion over time

C reduce gravity, slowing expansion over time

D reduce gravity, speeding up expansion over time

Expansion Archaeology

Goal: measure expansion rate at past times

Strategy:

we have seen: Hubble's law $v = H\ r$: always true
even though v, H, r all change over time
but as you look at distant objects
light travel time becomes large ("time machine effect")
$\rightarrow$ no longer probe present-day expansion rate
$H_{\text{today}} = H(t = t_0) = H(z = 0) = H_0$
but rather expansion in past $H(t_{\text{past}}) = H(\text{high} - z)$
$\Rightarrow$ Can use this to get expansion history
key requirement: need distance r to high-z objects

Q: what techniques are available?

Supernovae and Cosmodynamics

goal: measure expansion out to high z

key tool: **standard candle** $\equiv$ object of known L
measure flux F, then $d_{\rm lum} = \sqrt{F/4\pi L}$

need objects which:

- have fixed L indep of z, environment
- can see at high $z \rightarrow$ high $L \rightarrow$ supernova explosions

- Massive stars $\rightarrow$ SN: Type II
  bright, but: L varies w/ mass, metallicity X bad idea!
- SN Type Ia: exploding white dwarf
  WD $\rightarrow$ fixed mass of ^{56}Ni (radioactive) $\rightarrow$ ^{56}Fe
  decay sets $L(t) \rightarrow$ std candle!
  `www: SN 1994D`

iClicker Poll: Past Expansion Rate

in the real universe, with gravity

How should the expansion rate change with time?

A expansion slows with time $\rightarrow$ faster in past

B expansion constant with time $\rightarrow$ same in past

C expansion speeds up with time $\rightarrow$ slower in past

SN Ia and Expansion History

Pop fly Analogy

galaxies moving apart today $\leftrightarrow$ upgoing ball
would coast forever, but gravity attractive
slowing galaxies $\leftrightarrow$ slowing ball
$\rightarrow$ faster in past

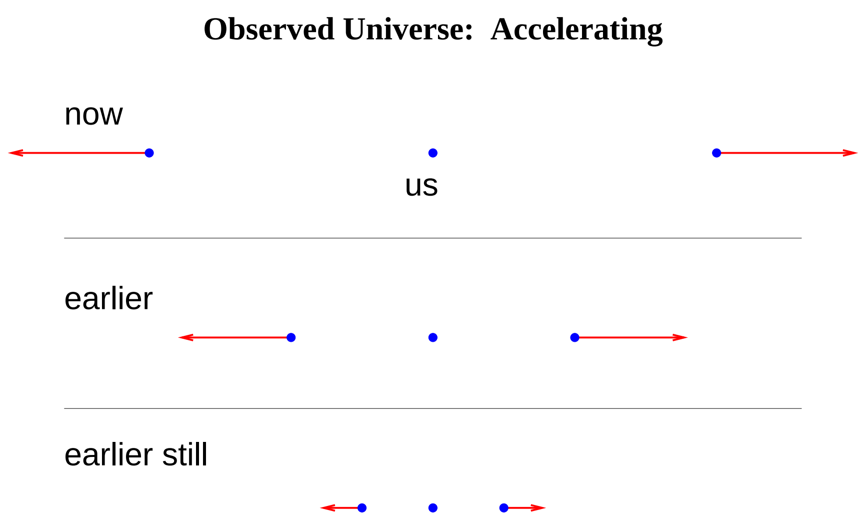
SN Data: distance indicator ($d_{\rm lum}(z)$ "luminosity distance")
traces expansion history $H(z) = \dot{a}/a$

- but expansion $H = \frac{\text{rate of change in } a}{a}$: a kind of "velocity"
  so variation in $H \rightarrow$ change in "velocity"
- H history measures cosmic acceleration

expectations: acceleration $= \ddot{a}/a = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right)$
$\Rightarrow \ddot{a} < 0$ gravity slows expansion

Distant Supernovae: The Verdict

Our actual observed universe: galaxies *slower* in past!



SN data: $H(z)$ **smaller** in the past (high z and small t)

$\Rightarrow H(z)$ **increases** with time!

$\Rightarrow \ddot{a} > 0$!

expansion rate: **accelerated!**

Q: what would this mean in the pop fly analogy?

Accelerating Universe: Pop Fly Analogy

Pop fly: ball thrown up in the air

ordinary baseballs: made of matter, feel Earth's gravity

$\rightarrow$ moves ever slower on the way up

$\rightarrow$ decelerated

but the *Universe* does the opposite!

a pop fly acting like the Universe

would get *faster* as it gets higher!

and so would launch itself to space!?!

Director's Cut Extras

Radiation Density and Cosmic Expansion

radiation = stuff moving at or near speed c

relativistic: $v \sim c$, $kT >> mc^2$

if particles not created/destroyed,

then total number const

• number density $n = N/V \propto a^{-3}$

energy density $\varepsilon_\gamma = E/V = E_\gamma n_\gamma$

where E_γ is the average energy of *one* particle

but we saw for photons: $E_\gamma \propto 1/a$

$\rightarrow \varepsilon_\gamma \propto a^{-4} \propto T^4$ if thermal but mass-energy equivalence $E = mc^2$ says that

$$m = \frac{E}{c^2} \tag{5}$$

$$\frac{m}{V} = \frac{E}{V}\frac{1}{c^2} \tag{6}$$

$$\rho = \frac{\varepsilon}{c^2} \tag{7}$$

and so

$$\boxed{\rho_{\rm rad} = \varepsilon_{\rm rad}/c^2 \propto a^{-4}} \tag{8}$$