

Astro 350  
Lecture 28  
Nov. 5, 2012

Announcements:

- **Homework 8** due at start of class Friday
- **Discussion 8** due Wednesday

Before exam: cosmic acceleration and the preposterous universe

*Q: what's so bizarre about the universe is accelerating?*

*Q: what are the possible explanations for this?*

*Q: which solution is the least radical?*

Man, there's a lot of unexplained phenomenon  
out there in the world.

Lot of things people say

What the heck's going on?

– Cosmologist Mojo Nixon

Cosmo-tip: acceleration is strange!

Expanding universe already a challenge to imagine  
...now add acceleration on top of this!

if it doesn't bother you

you probably need to think harder about it!

**Simplest solution:** to cosmic acceleration  
Einstein “cosmological constant”  $\Lambda$  (greek: Lambda)

originally invented by Big Al (1917):

- “fudge factor” in General Relativity
- invented to prevent cosmic expansion  
recall: Hubble’s law published in 1929  
stock market crashing, Universe expanding

cosmo constant  $\Lambda$  changes Newton’s gravity force law:  
mass  $m$  at distance  $r$  from mass  $M$  feels force

$$F_{\text{gravity}} = -\frac{GMm}{r^2} + \frac{1}{3}\Lambda mr \quad (1)$$

Q: what if  $\Lambda = 0$  and  $M = 0$ ? what does this mean?

Q: what do we get if  $\Lambda = 0$  but  $M > 0$ ? why the  $-$  sign?  
what happens to particle released from rest?

$\omega$  Q: what if  $M = 0$  and  $\Lambda > 0$ ?  
what happens to particle released from rest?

Q:  $\Lambda$  invented to prevent cosmic expansion—how?

In cosmological context,  $\Lambda$  changes acceleration to

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda \quad (2)$$

can “fine-tune”  $\Lambda$  to prevent expansion/collapse by exactly compensating for normal gravity and make the universe not accelerate or decelerate

*Q: how? what fine tuning needed?*

Einstein did this

but: after Hubble sees expansion in 1929,

Al allegedly sez  $\Lambda$  his “greatest blunder”

‡ *Q: what if  $\Lambda$  exist but not fine tuned?*

However: if  $\Lambda$  *not* fine-tuned  
can “overcompensate” for gravity attraction  
→ lead to acceleration!  
so: in wake of SNIa results,  $\Lambda$  rebirth!  
  
→ even AI’s blunders turn into gold!

in math language:  $\Lambda$  alters Friedmann:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} + \frac{\Lambda c^2}{3} = \frac{8\pi}{3}G\rho_{\text{tot}} - \frac{K}{a^2} \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho + 3\frac{P}{c^2}\right) + \frac{\Lambda c^2}{3} = -\frac{4\pi}{3}G\left(\rho_{\text{tot}} + 3\frac{P_{\text{tot}}}{c^2}\right) \quad (4)$$

where  $\rho_{\text{tot}}$  includes  $\rho_{\Lambda} = \Lambda c^2/8\pi G$  “vacuum energy density”  
 $P_{\text{tot}}$  includes  $P_{\Lambda} = -\rho_{\Lambda}c^2$

In other words:

- $\Lambda \neq 0$  gives energy content to empty space!  
(“vacuum energy”)
- and vacuum energy has negative pressure!  
 $\Rightarrow$  fills the bill for acceleration!

## The Onset of Acceleration

recall: supernova data show that

- the universe is *accelerating* now and in “recent” past in blatant contradiction to expectations
- but in more distant past, universe was *decelerating*

How can we understand this?

Example: if cosmological constant  $\Lambda$ , then

$$\text{acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda \quad (5)$$

Q: Which term is bigger today?

Q: what about the past—what is  $a$  like at earlier times?

Q: How does matter density  $\rho$  change in the past?  $\Lambda$ ?

Q: and so what happens in the past? what about the future?

$$\text{acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda \quad (6)$$

today:

- U accelerating, so acceleration  $> 0$
- and thus positive  $\Lambda$  term  $>$  negative  $\rho$  term

in the past:

- scale factor  $a$  smaller (U is expanding!)
- $\Lambda$  same, but matter density  $\rho = \rho_0/a^3 \propto 1/a^3$   
 $\Rightarrow$  in past, density higher:  $\rho$  bigger
- at some point in past (some value of  $a$ ):  
 negative  $\rho$  term wins  $\rightarrow$  U decelerates!
- in fact, not long ago: terms equal when  $a = 1/1.3 = 0.75!$

in the future:

- $\rho$  keeps getting smaller, but  $\Lambda$  same
  - acceleration becomes even more positive
- $\Rightarrow$  *a  $\Lambda$  universe will accelerate (and expand) forever!*

# $\Lambda$ and the Cosmic Coincidence Problem

cosmo constant  $\Lambda$  – what *is* it?

- a new constant of nature (like  $c$ ,  $G$ )
- can be viewed as “antigravity” source everywhere
- but also can be viewed as a “substance”  
filling all of space, at all times,  
with uniform density of energy  $\rho_\Lambda = \Lambda/8\pi G$   
*“vacuum energy density”*

curious fact: today  $\rho_\Lambda \approx 2\rho_{\text{matter},0}$

but  $\rho_\Lambda$  never changes with  $U$  expansion

while  $\rho_{\text{matter}}$  always changes

$\Rightarrow$  so why do we live at a special time:

almost at the moment when the two are equal?

at most cosmic times, either  $\rho_{\text{matter}} \gg \rho_\Lambda$

◦ or  $\rho_\Lambda \gg \rho_{\text{matter}}$

huge coincidence!?! seems anti-Copernican! *Q: ways out?*

# Dark Energy

to keep “spirit of  $\Lambda$ ”

but avoid cosmic coincidence problem:

generalize vacuum energy idea  $\Rightarrow$  **dark energy**

- previously unknown energy field (“scalar field”)  
known matter and energy fields fail!  
have positive pressure and thus attractive gravity
- also has negative pressure, causes acceleration
- but now density can change – and usually does!  
 $\rho_{\text{DE}}$  can drop with expansion  
but in some models can even *increase!*

## Dark Energy and Cosmic Coincidence

some dark energy models (“quintessence”)  
find dark energy change (evolution) is linked  
to the rest of cosmic contents (matter, radiation)

evolution occurs in such a way that  
always keep  $\rho_{\text{DE}}$  close  $\rho_{\text{matter+rad}}$   
so this is *always* true, not just now  
→ alleviates cosmic coincidence  
of acceleration starting “yesterday”

## Dark Energy vs Cosmo Constant

technically: dark energy density  $\rho_{\text{DE}} \propto a^{-3(1+w)}$

with  $w$  unknown except that need  $w < -1/3$  for acceleration

cosmological constant:  $w = -1$  exactly, so  $\rho_{\text{DE}} \propto a^0 = \text{const}$

Note: cosmo constant is *special case* of dark energy  
simplest possible version: unchanging always

*Q: so how do we tell if we have  $\Lambda$   
or more general dark energy?*

*Q: and who cares? what's the difference?*

# Unmasking Dark Energy

cosmo constant is very special:  
 $\Lambda$  and thus  $\rho_\Lambda$  strictly constant  
never change in time or space

so if can measure cosmic expansion in past  
can find the density needed to cause this  
see if it changes or not

technically: measure  $w$  from  $\rho_{\text{DE}} \propto a^{3(1+w)}$

cosmo constant if and only if  $w = -1$

Who cares?

- if  $\Lambda$ : why do we live at the moment  
it has revealed itself? Anthropic principle?
- if *not*  $\Lambda$ : what is this weird  
evolving dark energy that fills the universe?

## iClicker Poll: Refine Your Bets on Cosmic Acceleration

What is causing cosmic acceleration?

- A** a cosmological constant  $\Lambda$
- B** dark energy (but not special case of cosmo constant)
- C** modified gravity

# Uh Oh.

Warning!

some dark energy models have  $\rho_{\text{DE}}$  *increasing* with time!

i.e., expansion  $\rightarrow$  *larger* density of dark energy!?!

leads to scale factor growth

$$a_{\text{future}}(t) \propto \frac{1}{t_{\text{rip}} - t} \quad (7)$$

where  $t_{\text{rip}} > t_0$  is a fixed future time

*Q: what happens when  $t = t_{\text{rip}}$ ? why is this bad?*

## The Big Rip

if DE density increases with expansion  
then expansion rate  $H \propto \rho_{\text{DE}}$  accelerates more & more

$$a_{\text{future}}(t) \propto \frac{1}{t_{\text{rip}} - t} \quad (8)$$

*diagram: plot of  $a$  vs  $t$*

- ▷ scale factor  $a \rightarrow \infty$  at  $t = t_{\text{rip}}$
- ▷ in finite time, all particles move infinitely far from all other particles: **the big rip**

## Cosmologists & Ghostbusters Harold Ramis & Bill Murray

Harold: It would be bad.

Bill: I'm a little fuzzy on the whole  
"good/bad" thing here. What do you mean "bad"?

Harold: Try to imagine all life as you know it  
stopping instantaneously and every molecule  
in your body exploding at the speed of light.

www: Cosmologist Woody Allen, *Annie Hall* (1977)