Astronomy 406 **Fall 2007**

MIDTERM EXAM

October 26, 2007

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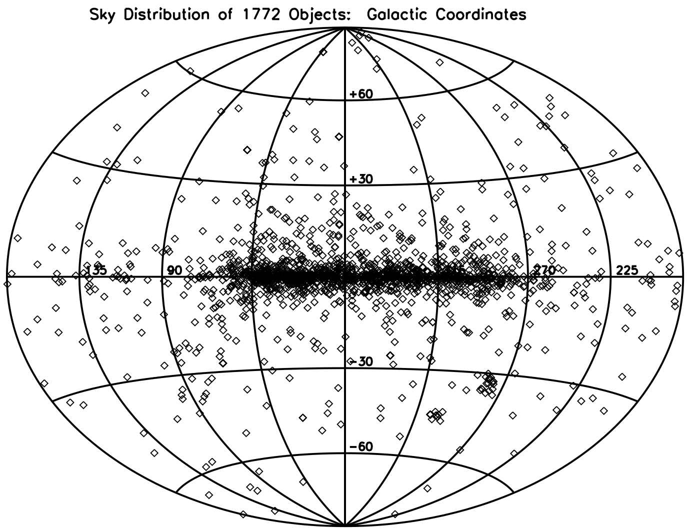
1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
2. Write you name and all answers in your test booklet. Turn in your booklet and this sheet.
3. Show all of your work in the test booklet, and indicate clearly your final answer! A correct final answer may not receive credit if no work is shown.
4. Budget your time! Don't get stalled on any one question.
5. The exam is open book and open notes.
6. For your reference there are constants listed below.
7. The total number of points on the exam is 120.

Possibly Useful Constants

Astronomical Unit: $1\,\mathrm{AU} = 1.5 \times 10^{11}\,\mathrm{m}$.
parsec: $1\,\mathrm{pc} = 3.1 \times 10^{16}\,\mathrm{m} = 2.1 \times 10^{5}\,\mathrm{AU}$
gravitational constant : $G = 6.7 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} = 4.5 \times 10^{-3}\ \mathrm{pc\ km^2\,s^{-2}}\ {M_\odot}^{-1}$
speed of light: $c = 3.0 \times 10^{8}\ \mathrm{m\,s^{-1}} = 3.0 \times 10^{5}\ \mathrm{km\,s^{-1}} = 1.023\ \mathrm{pc\,Myr^{-1}}$
Stefan-Boltzmann constant: $\sigma = 5.7 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$
Wien's Law constant: $b = 2.9 \times 10^{-3}\ \mathrm{m\ K}$
Planck's constant: $h = 6.6 \times 10^{-34}\ \mathrm{J\ s} = 4.1 \times 10^{-15}\ \mathrm{eV\ s}$
electron Volt: $1\ \mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$
Boltzmann constant: $k = 1.4 \times 10^{-23}\ \mathrm{J\ K^{-1}} = 8.6 \times 10^{-5}\mathrm{eV\ K^{-1}}$
mass of the proton: $m_p = 1.7 \times 10^{-27}\,\mathrm{kg} = 0.938\,\mathrm{MeV}/c^2$
Hubble constant (present-day value): $H_0 \simeq 73\ \mathrm{km\ s^{-1}\ Mpc^{-1}} = 2.3 \times 10^{-18}\ \mathrm{s^{-1}}$
Hubble length: $d_H = c/H_0 = 4.1 \times 10^{3}\ \mathrm{Mpc}$
solar mass: $M_\odot = 2.0 \times 10^{30}\ \mathrm{kg}$
solar (total) luminosity: $L_\odot = 3.8 \times 10^{26}\,\mathrm{Watt} = 3.8 \times 10^{33}\ \mathrm{erg\,s^{-1}}$
present age of Sun $\sim 5 \times 10^{9}$ yr

1. *Mystery Radio Sources.* A certain class of objects (which you have heard of) appear as point sources in radio wavelengths. To date, a total of 1772 of these objects have been detected in the radio. The figure below displays their positions on the sky, plotted in Galactic coordinates.

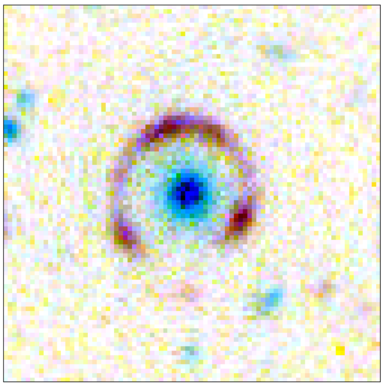


   (a) [10 points] Based on the sky distribution, are these objects dominantly Galactic or extragalactic? Briefly explain your reasoning.

   (b) [15 points] Based on the sky distribution, does our sample span the entire region occupied by these objects, or just the local neighborhood? That is, if they are Galactic, are we seeing them throughout the entire Galaxy? If they are extragalactic, are we seeing them beyond the local supercluster? Briefly explain.

   (c) [15 points] The radio emission from each of these objects is periodic, with periods ranging down to $P \approx 10^{-3}$ s $= 1$ ms. Assume that these objects are spinning spheres of mass M, radius R, and uniform density ρ. If we assume that the fastest-spinning of these objects are barely gravitationally bound, this means that their gravitational timescale has $\tau_{\rm grav} \approx P$. Use this to estimate the objects' density ρ. Finally, use the result to speculate as to what these mystery objects might be.

2. *An Einstein Ring.* Below an image of a nearly-perfect Einstein ring around a foreground galaxy. The foreground galaxy ("lens") has redshift $z_{\rm lens} = 0.44$, and the ring is a distorted background galaxy ("source") with redshift $z_{\rm source} = 2.4$. The ring observed to have an angular radius $\theta_{\rm E} = 5$ arc sec $= 10^{-4}$ radian.



   (a) [10 points] Using Hubble's law, find the distances $d_{\rm source}$ and $d_{\rm lens}$ to the two galaxies. Express your answers in Mpc.

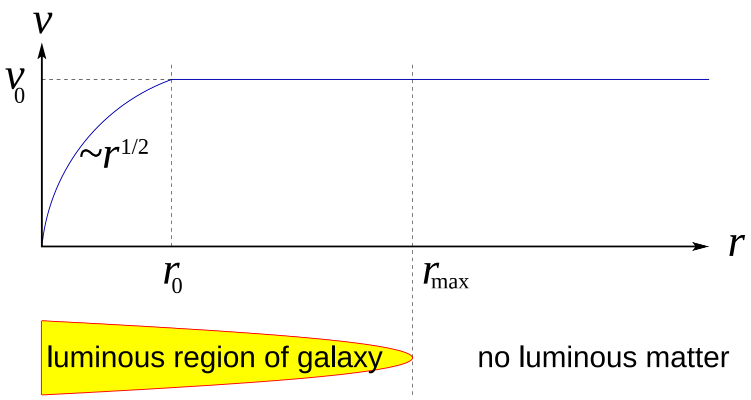
   (b) [10 points] Use the angular Einstein radius to calculate the physical radius $r_{\rm E}$ of the Einstein ring; express your answer in kpc. Compare this to the size of the luminous parts of a typical galaxy. Do you expect the lens mass to include dark matter in the lens galaxy? Briefly explain.

   (c) [15 points] Calculate the mass of the lens galaxy. Express your answer in terms of $M_\odot$ (i.e., find $M_{\rm lens}/M_\odot$). How does your result compare with the appropriate Milky Way mass?

3. *A Galaxy Rotation Curve.* A spiral galaxy is seen edge-on, and is measured to have the following rotation curve of orbit speed v versus galactocentric distance r:

$$v(r) = \begin{cases} v_0 \ (r/r_0)^{1/2} & r \leq r_0 \text{ (inner galaxy)} \\ v_0 & r > r_0 \text{ (outer galaxy)} \end{cases} \tag{1}$$

i.e., v increases as $v \propto \sqrt{r}$ until $r = r_0$ beyond which v is constant at v_0. See also the plot below



You may assume the disk material has circular orbits and is subject to the gravitation of a spherical mass distribution.

(a) [15 points] Find an expression for the enclosed mass $m(r)$ for all r. Briefly comment its the behavior $r \to 0$ and $r \to \infty$.

(b) [15 points] Beyond a distance $r > r_{\rm max}$ in the flat region of the rotation curve, the galaxy's luminous matter has $\rho_{\rm lum}(r) = 0$, as seen in the figure. Briefly explain why this result is evidence for dark matter.

(c) [15 points] Instead of dark matter, it is possible that gravity behaves in an unexpected (i.e., not Newtonian/Einsteinian) manner on these scales. Focus on the non-luminous region $r > r_{\rm max}$ of the flat rotation curve $v = v_0$, but now assume there is *no* matter present in this region. In this case, find the needed r-dependence of the modified (non-Newtonian) gravitational field $g_{\rm mod}(r)$. That is, writing $g_{\rm mod} \propto 1/r^{\beta}$, find β.