Astronomy 406 **Fall 2007**

MIDTERM EXAM–SOLUTIONS

---

1. *Mystery Radio Sources*

   (a) A Galactic population would trace the Galactic plane, while an extragalactic population would be unrelated to the Galactic plane. The mystery objects overwhelmingly lie in the Galactic plane, and thus they must be a Galactic population.

   (b) The objects are observed in radio wavelengths, and the Galaxy is transparent at these wavelenghts. Thus it is *possible* to see these objects throughout the entire Galaxy, if they are bright enough. If we only could see those nearby, then the situation would be like the optical observations of stars–we would see a uniform band encircling the Galactic plane. But these objects are not uniform, and instead are very much concentrated towards the inner Galaxy–just as in the case of 21 cm from neutral hydrogen, and CO molecular observations. Hence we conclude that (like neutral H and CO), these objects are seen throughout the Galaxy.

   (c) As shown in PS 1, the gravitational timescale for objects of density ρ is

   $$\tau_{\rm grav} = \frac{1}{\sqrt{G\rho}} \tag{1}$$

   If we put $P \approx \tau_{\rm grav} = 1/\sqrt{G\rho}$, then solving for density we have

   $$\rho = \frac{1}{GP^2} = 1.5 \times 10^{13}\ {\rm g/cm^3} = 1.5 \times 10^{16}\ {\rm kg/m^3} \tag{2}$$

   This enormous density, which demands that these are some sort of compact object. Reasonable guesses would be white dwarfs, or better, neutron stars.

   In fact, these objects are spinnin neutron stars–they are pulsars.

2. *An Einstein Ring.*

(a) Hubble's law is $v = H_0 d$, and using $v = cz$, we have

$$d = \frac{v}{H_0} = \frac{c}{H_0} z = 4100 \text{ Mpc } z \tag{3}$$

So we have

$$d_{\text{source}} = \frac{c}{H_0} z_{\text{souce}} = 9800 \text{ Mpc} \tag{4}$$

$$d_{\text{lens}} = \frac{c}{H_0} z_{\text{souce}} = 1800 \text{ Mpc} \tag{5}$$

(b) For a small angle such as this, we have $r = d\theta$. There was a stupid typo in the exam, since 5 arsec $= 2 \times 10^{-4}$ rad, rather than 10^{-4} rad, so you either got

$$r_{\text{E}}(5 \text{ arsec}) = d_{\text{lens}} \theta_{\text{E}} = 0.180 \text{ Mpc} = 180 \text{ kpc} \tag{6}$$

or

$$r_{\text{E}}(10^{-4} \text{ rad}) = \text{d}_{\text{lens}} \theta_{\text{E}} = 0.044 \text{ Mpc} = 44 \text{ kpc} \tag{7}$$

Either got full credit.

The luminous parts of a typical galaxy, such as the Milky Way, extend to a radius of about 10 kpc. Since r_{E} is considerably larger (for either value), we expect this lensing mass to extend into the dark halo and thus enclose dark matter as well as luminous matter.

(c) From lecture notes, or from the textbook, we see that

$$\theta_{\text{E}} = \sqrt{\frac{4GM_{\text{enc}}}{c^2 D_{\text{eff}}}} \tag{8}$$

where

$$D_{\text{eff}} = \frac{d_{\text{source}} d_{\text{lens}}}{d_{\text{source}-\text{lens}}} = \frac{d_{\text{source}} d_{\text{lens}}}{d_{\text{source}} - d_{\text{lens}}} = 2200 \text{ Mpc} \tag{9}$$

Using these we can solve for the enclosed mass

$$M_{\text{enc}} = \frac{\theta_{\text{E}}^2 c^2 D_{\text{eff}}}{4G} = 1.4 \times 10^{46} \text{ g} = 6.8 \times 10^{12} \ M_{\odot} \tag{10}$$

or, if you used the other value of θ_{E},

$$M_{\text{enc}} = 2.3 \times 10^{47} \text{ g} = 1.2 \times 10^{14} \ M_{\odot} \tag{11}$$

The Milky Way mass out to 50 kpc, including the dark halo, is about $5 \times 10^{11} M_{\odot}$, so this galaxy is in either case considerably more massive.

3. *A Galaxy Rotation Curve*

(a) For circular orbits, we have

$$\frac{v^2}{r} = \frac{G m_{\rm enc}(r)}{r^2} \tag{12}$$

and so we find

$$m_{\rm enc}(r) = \frac{v(r)^2\, r}{G} \tag{13}$$

Applying this to the rotation curve at hand, we havve

$$m_{\rm enc}(r) = \begin{cases} \frac{v_0^2 r^2/r_0}{G} & r \le r_0 \text{ (inner galaxy)} \\ \frac{v_0^2 r}{G} & r > r_0 \text{ (outer galaxy)} \end{cases} \tag{14}$$

We see that for $r \to 0$, that $m_{\rm enc} \propto r^2 \to 0$; the enclosed mass goes to zero at the origin, as it should. At large distances $r \to \infty$, we have $m_{\rm enc}(r) \propto r \to \infty$; this is the usual flat-rotation-curve result that the mass keeps increasing at large distances even though we don't see any luminous matter.

(b) Since there is no luminous matter beyond $r_{\rm max}$, but we find that $m_{\rm enc}(r)$ keeps increasing beyond $r_{\rm max}$, we conclude that there must be large amounts of non-luminous matter present. This is dark matter.

(c) We are still looking at circular motion, so we still want gravity to provide the centripetal acceleration, but now we will allow g to be non-newtonian. Thus we have

$$a_{\rm centripetal} = \frac{v^2}{r} = g_{\rm mod}(r) \tag{15}$$

and on the flat part of the rotation curve, $v(r) = v_0 =$ a constant, so we have

$$g_{\rm mod}(r) = \frac{v_0^2}{r} \propto \frac{1}{r} \tag{16}$$

and thus if we write $g(r) \propto 1/r^{\beta}$, we find that $\beta = 1$.