Astronomy 406, Fall 2013
Problem Set #12: The Final Frontier
Due in class: Friday, Dec. 6   Total Points: 60 + 5 bonus

1. *BBN and the cosmic baryon inventory.* In class it was asserted that the baryon-to-photon ratio $\eta \equiv n_{\rm baryon}/n_\gamma$ is proportional to the cosmic baryon density parameter $\eta \propto \Omega_{\rm baryon} h^2$. The BBN determination of η is thus also a measurement of $\Omega_{\rm baryon} h^2$ and hence the cosmic baryon content.

   (a) [5 points] Show that the present baryon mass density $\rho_{\rm baryon,0} \propto \eta n_{\gamma,0}$, and give the constant of proportionality (for the purposes of the calculation, you may take protons and neutrons to have the same mass m_p).

   (b) [5 points] Go on to:

      i. Show that $\eta \propto \Omega_{\rm baryon} h^2$, where $H_0 = 100\ h\ {\rm km\ s^{-1}\ Mpc^{-1}}$ defines the annoying "little h" factor the encodes our ignorance of the Hubble constant (in fact, $h \approx 0.7$, but keep you answer in terms of arbitrary h).

      ii. Find the constant of proportionality between η and $\Omega_{\rm baryon} h^2$, and evaluate it using $n_{\gamma,0} = 411\ {\rm cm}^{-3}$.

      iii. You should find $\Omega_{\rm baryon} h^2 \approx 3.6 \times 10^7 \eta$. The CMB gives the best current measure of the baryon-to-photon ratio, $\eta = 6.04 \times 10^{-10}$, and we also have $h \approx 0.7$. What is $\Omega_{\rm baryon}$ in this case?

2. *Where have all the baryons gone?* With the results from BBN under our belts, we are in a position to come full circle and make connections with our discussions of dark matter from the beginning of the course.

   (a) [5 points] Briefly explain how we know from BBN that most of the baryons in the universe are dark.

   (b) [5 points] Galaxy clusters are special objects because they are thought to contain a fair sample of cosmic material (although clusters themselves are rare and make up a small portion of cosmic matter). In clusters, where are baryons found? What does this suggest as a possible "hiding place" of dark cosmic baryons?

   (c) [5 points] In our initial considerations of dark matter in the Milky Way, we looked in detail at another possible candidate for dark baryons. What are these objects, and what do current data on these objects imply for their status as a "reservoir" of dark baryons?

3. *How big is the universe?* The true total volume of the universe depends on the value of κ and hence on whether the universe is closed (and thus finite) versus open or critical (and infinite). However, it is a separate issue as to what is the volume of the universe which is *observable* at any given time t.

   (a) [5 points] Argue that a rough estimate of the size of the observable universe today is ct_0.

   (b) [5 points] Using the results from Problem Set 10, argue that the true size of the observable universe is $\ell_{\rm hor}(t_0)$.

(c) [5 points] Go on to compute the size of the observable universe today, using $t_0 = 14$ Gyr and assuming that the universe is matter-dominated. Express your answer in Mpc.

(d) [5 points] Using your result from part (b), and your result above for $\Omega_{\rm baryon}$ (using $h \approx 0.7$), compute the number of baryons in the observable universe.

For comparison, also compute the number of baryons in your body.

4. *Cosmic Structure Formation.* Consider a "toy" universe that is perfectly homogeneous, except for a spherical region of constant density *higher* than the cosmic mean density. To make matters simple, imagine that the universe outside of the overdense region has no dark energy, is flat ($\kappa = 0$), and has only matter, so $\Omega = \Omega_{\rm matter} = 1$.

   (a) [5 points] Show that in this simplified universe (outside of the overdense region) $\rho_{\rm universe} = \rho_{\rm crit}$ always, even though this value evolves with time.

   (b) [5 points] Now consider the overdense region, which has $\rho_{\rm region} > \rho_{\rm universe}$. Imagine that it initially expands with the rest of the universe. What happens eventually?

      i. To see this, first explain why $\Omega > 1$ in this region.

      ii. Then recall that Gauss' law tells us we need only pay attention to the matter in a spherical region, In light of these fact, what do you expect the overdensity to do? Note that you are not asked for a calculation, but rather a physical argument based on the given facts.

      iii. Similarly, imagine a spherical *underdensity* $\rho_{\rm region} < \rho_{\rm universe}$. What do you expect the fate of this region to be?

   (c) [5 points] Now turn to the real universe. Recall that the CMB tells us that the early universe contained fluctuations in the matter density, with both overdensities and underdensities compared to the cosmic average density. How do you expect these fluctuations to evolve over time? What does this imply for the universe today and in the future?

5. [5 **bonus** points] *Proofreading The Galaxy Song.* Below are the lyrics to "The Galaxy Song," featured in the Monty Python cinematic classic, *The Meaning of Life.* Aside from being a toe-tappin' good tune, it has the scientific virtue of being quantitatively correct. That is, all of the numbers in it are at least arguably good to within a factor of 2.

   Except two.

   For 5 big bonus points:

   (a) Find *both* numbers that are clearly incorrect.

   (b) Give the correct value for each of these number, in the units mentioned in the song.

   (c) Comment on the astrophysical implications if the stated number *were* correct.

   Note that the bonus points are for finding a quantitative error; there are a couple of conceptual points that are at least debatable, but the points are for a specific numerical error.

“The Galaxy Song”
Python, M. (1987), *The Meaning of Life*

Whenever life gets you down, Mrs. Brown,
And things seem hard or tough,
And people are stupid, obnoxious or daft,
And you feel that you’ve had quite eno-o-o-o-ough,

Just remember that you’re standing on a planet that’s evolving
And revolving at nine hundred miles an hour.
It’s orbiting at nineteen miles a second, so it’s reckoned,
’Round the sun that is the source of all our power.
Now the sun, and you and me, and all the stars that we can see,
Are moving at a million miles a day,
In the outer spiral arm, at forty thousand miles an hour,
Of a Galaxy we call the Milky Way.

Our Galaxy itself contains a hundred billion stars;
It’s a hundred thousand light-years side to side;
It bulges in the middle sixteen thousand light-years thick,
But out by us it’s just three thousand light-years wide.
We’re thirty thousand light-years from Galactic Central Point,
We go ’round every two hundred million years;
And our Galaxy itself is one of millions of billions
In this amazing and expanding universe.

Our universe itself keeps on expanding and expanding,
In all of the directions it can whiz;
As fast as it can go, that’s the speed of light, you know,
Twelve million miles a minute and that’s the fastest speed there is.
So remember, when you’re feeling very small and insecure,
How amazingly unlikely is your birth;
And pray that there’s intelligent life somewhere out in space,
’Cause there’s bugger all down here on Earth!