Astronomy 406, Fall 2013
Problem Set #3

Due in class: Friday, Sept. 20; Total Points: 60 + 5 bonus

1. *Getting to know your neighbors–the local stellar neighborhood.* The course `links` directs you to webpages on the 170 brightest stars and the 100 nearest stars.

   (a) [4 points] Using parallax values in the RECONS table of the 100 nearest stars, calculate the distance, in parsecs and in light-years, of the members of the nearest star system, α Centauri, also known as Rigil Kentaurus. This is a three-star system composed of Proxima Centauri, α Centauri A, and α Centauri B. Why is Proxima Centauri so named?

   The Sun has an *apparent* visual magnitude of $m_V \equiv V = -26.75$. Calculate the apparent magnitude of the Sun at the distance of α Centauri A. Then use the tables to answer: Which is more luminous, the Sun or α Cen A? If α Cen A had the Sun's luminosity, would it be among the 125 brightest stars in the sky? If so, where?

   (b) [4 points] Consider the 100 nearest star systems, as tabulated by the RECONS project. For number 100, calculate the distance r in pc. Then find the volume of a sphere with this radius, expressed in units of pc^3. Use this to compute the average number density $n_\star$, i.e., number per unit volume, of the 100 nearest stars, in units of $\mathrm{stars/pc^3}$.

   Note: this is the number density of star *systems*; based on the data in the table, how much higher would you estimate the number density of individual *stars* to be?

   Now imagine the nearby stars are evenly spread in space (e.g., in a cubic lattice). In this case, show that distance between nearest neighbors is $\ell_\star = n_\star^{-1/3}$, where $n_\star$ is the number density of stars.

   Go on to compute this number given your result from part (a), and comment.

   (c) [4 points] At the bottom of the page in the RECONS tabulation you will find the number of stars and of star systems within 10 pc of the Sun. Use this to estimate $n_\star$ and $\ell_\star$, and compare your answer to those of part (c).

   Finally, compare both of your estimates for $n_\star$ with the professional result gotten from summing up all stars in the luminosity function in Figure 2.3 of Sparke & Gallagher; this result is appears on p. 64.

   (d) [4 points] Of the 100 nearest stars: What are the highest and lowest masses? What trends do you see in the masses of nearby stars? How does the Sun compare to its nearest neighbors?

   For the 100 nearest stars, what are the highest and lowest absolute visual magnitudes M_V (called `Mv` in the table)? What trends do you see in the absolute magnitudes of nearby stars? How does the Sun compare to its nearest neighbors?

   Finally, compare these results with the trends illustrated in Figure 2.3 of Sparke & Gallagher. Are your findings in line with what is seen in the plot?

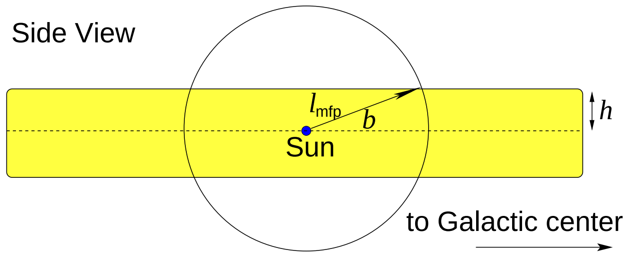
   (e) [5 **bonus** points] If stars are uniformly distributed with a number density $n_\star$, find an expression for the total ("cumulative") number of stars within a volume of radius r; this is denoted $N(< r)$, and you should find a strong dependence on r.

Using one of your observed $n_\star$ values, plot the expected $N(< r)$ behavior over the range $r = 0$ to 10 pc. I recommend using a computer plotting routine to do this.

Now use the actual data on the nearest stars to find the observed cumulative number $N_{\rm obs}(< r)$. How well does your predicted smooth curve fit the data? How good is the "uniform density" approximation used to compute the curve?

2. *Optical Star Counts as Probes of the Stellar Disk.* On the course `links` webpage, there is a link to the *Tycho-2* website and an image summarizing its main result. *Tycho-2* was part of a space mission which make precision measurements of the 2.5 million (!) brightest stars in the sky, in the *optical* wavebands B and V. The sky plot of their results clearly shows the stellar disk, as well as the effects of dust.

   We will use these data in the context of a simple model for the Galactic stellar disk; see sketch at right. We will assume that the number density $n_\star$ of stars is uniform and constant throughout the disk, which we will take to have a (cylindrical) radius $R_{\rm disk} \sim 15$ kpc, with the sun at Galactocentric radius $R_\odot = 8$ kpc. We will also assume that the disk height (from the midplane which is the dotted line in the sketch) is h, i.e., the total disk thickness is $2h$.



   (a) [5 points] In Problem Set 2, you calculated the mean free path $\ell_{\rm mfp}$ for optical photons in the dusty interstellar medium. In reality, we measure $\ell_{\rm mfp}$ using observations of stars. Consider a star at distance r, which *would have* flux F_0 and apparent magnitude m_0 if there were no dust at all in the sightline. If there *is* dust, then we observe a flux F and apparent magnitude m. The effect of dust is thus encoded in the *difference* between the observed and unobscured magnitudes, a quantity known as the *extinction* $A = m - m_0$. Different wavebands have different extinctions.

   One can show (S&G eq. 1.21) that the observed flux is $F = F_0\, e^{-r/\ell_{\rm mfp}}$. Using this, show that the extinction obeys as $A = Kr$, that is, extinction grows with distance as $A \propto r$. Find the proportionality constant K in terms of $\ell_{\rm mfp}$.

   (b) [4 points] In the Galactic disk, typically $K \approx 1$ mag/kpc in this V band. Use this to find $\ell_{\rm mfp}$.

   Now calculate the ratio of the visible disk area (inside $\ell_{\rm mfp}$) to the total disk area. Comment on the implications of your result.

   Comment on the significance of the nearly uniform stellar density *Tycho-2* sees in the Galactic plane for all longitudes.

   (c) [5 points] The telescope on *Tycho-2* had a limiting apparent magnitude in the V band of about $m_{V,\rm lim} = 11.5$ mag. Consider stars at a distance (from us; see circle in sketch) of $r = \ell_{\rm mfp}$. Find the *absolute* magnitude $M_{V,\star}$ such stars must have for *Tycho-2* to see them, given the apparent magnitude limit. Then consult Tables 1.4–1.6 in Sparke & Gallagher, and comment on which types of star can be seen out to distances $r = \ell_{\rm mfp}$.

(d) [5 points] From the *Tycho-2* map in Galactic coordinates, we clearly see a fairly sharp dropoff in stars at some Galactic latitudes $|b|$ above (and below) the midplane. In the context of our simple model sketched above, explain how this immediately implies that the disk height $h < \ell_{\rm mfp}$, as was assumed in drawing the sketch.

Go on to make a quantitative estimate for h as follows. From the sky plot, estimate the Galactic latitude b at which we see upper edge of the disk. Use this value and some geometry based on the sketch above to compute h; express your answer in kpc. This is an estimate of the scale height of the stellar disk.

Compare your result to Sparke and Gallagher Figure 2.8, and comment.

(e) [5 points] Finally, we see that towards the Galactic poles (i.e., at $b = \pm 90°$) *Tycho-2* see about 25 stars/deg^2, i.e,. about $N_\star = 25$ stars in each patch of angular area $\Omega = 1° \times 1°$ on the sky. This is a projection of all stars out to a distance h within a narrow "cone" centered on us with angular area Ω. But one can show (you are welcome to, but don't have to!) that the 3-D volume of such a cone is $V_{\rm cone} = \Omega h^3/3$ (where Ω is expressed in square radians or "steradians": 1 rad$^2 \equiv 1$ sr). Use this with your value for h, the measured value of $N_\star$ in area $\Omega = 1° \times 1°$ (expressed in steradians), and compute the number density $n_\star$ of stars implied by the *Tycho-2* survey. Compare your result to the other estimates you have made of this quantity, and comment on the agreement.

3. *Gravity and Gauss' Law: a Planar Galaxy.* For a first approximation, we will simplify the Galactic disk to an infinite plane. However, we will not make it infinitely thin, but we'll assign a density $\rho(z)$ which is only a function of height z. We wish to know what life is like in this situation.

(a) [5 points] Use symmetry to argue that the gravitational field $\vec{g}$ can only point in the vertical (z) direction, and will only depend on z: $\vec{g}(\vec{r}) = g(z)\hat{z}$.

(b) [5 points] Further simplify to the case of constant density ρ_0 up to a height h:

$$\rho = \rho(z) = \begin{cases} \rho_0 & |z| \leq h \\ 0 & |z| > h \end{cases} \tag{1}$$

For this situation, use Gauss' law to find $g(z)$ for all z.

(c) [5 points] Now consider a star which is released at rest at $t = 0$ from height z_0, with $|z| < h$. Show that the motion of the star obeys the expression $\ddot{z} = -\omega^2 z$. Find and expression for ω in terms of ρ_0 and physical constants.

Find $z(t)$, and describe the motion of the star. Find the period P of the motion, in years, using a local mass density $\rho_0 \sim 0.03\ M_\odot\ {\rm pc}^{-3}$. Comment on the implications for the Sun's motion, and our view of the Galactic center over time.

Compare your answer to the gravitational timescale $\tau_{\rm grav}$ computed as in Problem Set 1, and comment.

(d) [5 points] Finally, consider a star released from rest at $t = 0$ from height $z_0 \gg h$. Show that the star experiences constant acceleration which depends only on the "surface density" $\Sigma = \int_{-h}^{h} \rho(z) dz$. Calculate how star speed v depends on height z in this case. Comment on how measurements of star vertical speeds can be used to estimate the surface density of the disk.