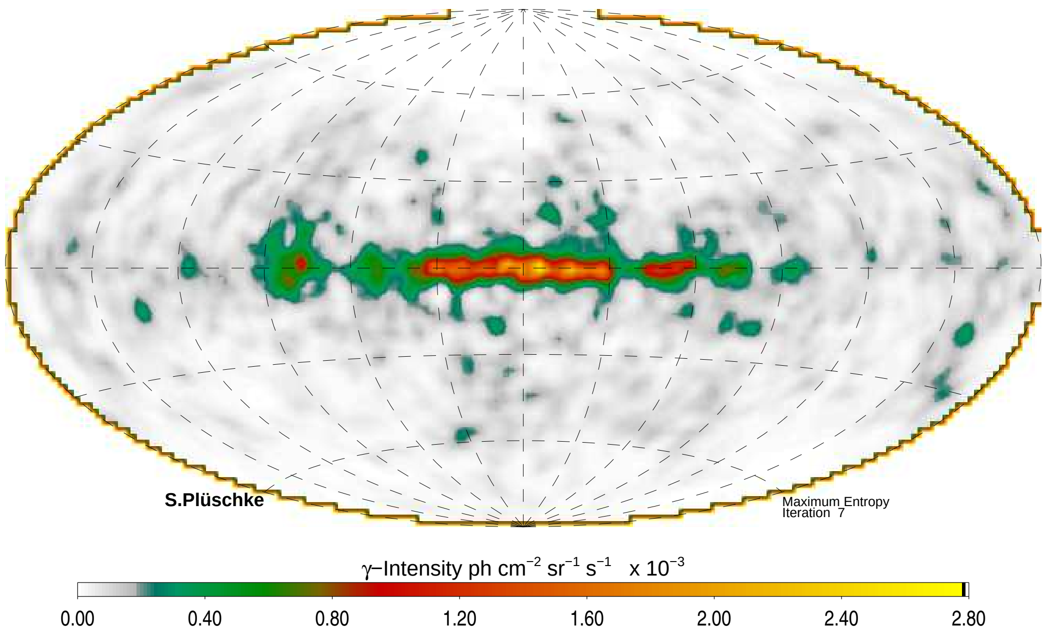
Astronomy 406, Fall 2013
Problem Set #4

Due in class: Friday, Sept. 27 ; Total Points: 60 + 5 bonus

1. *Our Radioactive Galaxy and the Galactic Supernova Rate.* Supernova explosions produce many heavy elements, including an unstable form (isotope) of aluminum, known as aluminum-26. ^{26}Al is radioactive but long-lived, a mean life $\tau(^{26}\mathrm{Al}) = 1$ Myr. Its decay produces a monoenergetic gamma ray of energy $\epsilon_\gamma = 1.809$ MeV. Below is an all-sky map at 1.809 MeV, as seen by the COMPTEL instrument on NASA's *Compton Gammma-Ray Observatory* space mission (Plüschke et al 2001; `arXiv:astro-ph/0104047`).



   (a) [5 points] Explain why the sky distribution of the 1.809 MeV signal suggests that our Galaxy is transparent to these photons.

   Hint: if the opposite were true, and gamma rays were absorbed in interstellar space with some mean free path, then what would you expect the gamma-ray sky distribution to look like?

   (b) [5 points] The COMPTEL map gives the γ-ray intensity or surface brightness I, i.e., the *number* of photons per unit area per unit time and per unit *angular area on the sky*; this angular area is given in steradians $\equiv$ sr = radian^2. To find the total γ-ray number flux F, very roughly *estimate* the integral $F = \int I \, d\Omega$ of the surface brightness over angular area. Express your results in photons cm^{-2} s^{-1}.

   Be careful with angular units! Also note that the intensity scale is given in units of 10^{-3} photons cm^{-2} s^{-1} sr^{-1}.

   (c) [5 points] Although the total ^{26}Al flux F comes from all across the Galactic disk, assume for simplicity it is all produced at the Galactic center, with a distance $d = R_0 \simeq 8.5$ kpc. Using this, estimate the ^{26}Al gamma-ray luminosity $L = \dot{\mathcal{N}}$, i.e., the number of decay photons per unit time. Express your answer in photons/sec.

(d) [5 points] The radioactive decay law says that $\dot{\mathcal{N}} = \mathcal{N}/\tau$, where τ is the mean life given above.

  i. Use this relation to find the number $\mathcal{N}$ of ^{26}Al atoms in the Galaxy today.

  ii. Then find the mass M of ^{26}Al in the Galaxy today, using the fact that one ^{26}Al atom has mass $m \approx 26m_p$ (with m_p the proton mass). Express your answer in solar masses $M_\odot$.

  iii. Compare your result to the total mass of gas in the Galaxy today, and comment on how radioactive the Galaxy is.

(e) [5 points] Supernova explosions produce ^{26}Al, each explosion typically ejecting a mass of $M_{\text{ej}} \sim 3 \times 10^{-5} M_\odot$ into the interstellar medium.

  i. Show that if the total rate of supernova explosions in the Galaxy (i.e., number of supernovae per unit time) is $\mathcal{R}_{\text{SN}}$, then the total rate at which ^{26}Al mass is produced is $M_{\text{ej}}\mathcal{R}_{\text{SN}}$.

  ii. Go on to show that if the ^{26}Al decay rate balances the ^{26}Al supernova production rate, then $M_{\text{ej}}\mathcal{R}_{\text{SN}} = m\dot{\mathcal{N}}$, where $m \approx 26m_p$ is the mass of one ^{26}Al atom.

(f) [5 points] Use your value for $\dot{\mathcal{N}}$ you can now find a value for the Galactic supernova rate $\mathcal{R}_{\text{SN}}$.

  i. Calculate $\mathcal{R}_{\text{SN}}$, and express your answer in events/year.

  ii. Use this result to calculate the amount of time τ_{SN} between supernova explosions anywhere in the Galaxy.

  iii. Based on your results, what do you think are the chances there will be a Galactic supernova in your lifetime? Why?

(g) [5 points] The most recent observed Galactic supernova exploded about $\tau_{\text{SN,obs}} =$ 400 years ago. You should find $\tau_{\text{SN,obs}} \gg \tau_{\text{SN}}$, i.e., the observed interval is significantly longer than your answer from the previous question. Explain this discrepancy (Hint: until the last few decades, only observations in the optical were available.)

(h) [5 **bonus** points] Imagine a supernova exploded today on the opposite side of the Galactic disk. Explain why it would not be detectable by the Hubble telescope. Then explain what observations (using present-day facilities) we could make to discover this supernova.

2. *The Distribution of Gas in the Galaxy: 21 cm Data.* In class we derived an expression for line-of-sight velocity $v_{\text{los}} = v_r$ and compared this with 21 cm data. This comparison can be pushed further by modeling the Galaxy as a single ring of material at radius R (not necessarily equal to our radius R_0), with angular velocity $\Omega(R)$ and circular speed $V(R) = \Omega(R)R$. Then the real Galaxy can be thought of as a sequence of these rings. (See also SG problem 2.15 and hints at the back of the book).

  (a) [5 points] Consider a ring interior to us (i.e., with $R < R_0$).

    i. Explain why v_{los} is maximum at the point at which the ring is tangent to our line of sight. A diagram may help here.

    ii. Also show that the latitudes of the two tangent points are $\ell_t = \pm \arcsin(R/R_0)$.

iii. Find $v_{\rm los}$ at these tangent points in terms of R and $V(R)$.

(b) [5 points] Again for a ring at $R < R_0$:

i. Give the exact expression for $v_{\rm los}$ for all ℓ between $\pm\ell_t$ (Hint: the class and text discussion is very relevant).

ii. Using this and taking $V(R) = V_0 = 220$ km/s everywhere, sketch how the 21-cm (l, v) plot (as in SG Fig. 2.18) would look for such a ring at $R = R_0/2$ and $R = R_0/10$.

(c) [5 points] Find the slope $dv/d\ell$ at $\ell = 0$ for your result in part (b).

If $\Omega(R)$ is decreasing, what do you learn if the observed slope is steep? shallow? Make sure your plots from (b) reflect this slope.

(d) [5 points] Now consider a ring exterior to us, $R > R_0$.

i. Find $v_{\rm los}$ as a function of ℓ for this case.

ii. Also find $dv/d\ell$ at $\ell = 0$.

iii. Sketch the (ℓ, v) plot for this case at $R = 1.5R_0$ and $R = 2R_0$.

(e) [5 points] Now look at the 21 cm (ℓ, v) map in SG Figure 2.20 and explain where the gas lies that corresponds to: ($\ell \sim 50°$, $V > 0$); ($\ell \sim 50°$, $V < 0$); ($\ell \sim 120°$, $V < 0$); ($\ell \sim 120°$, $V < 0$); ($\ell \sim -60°$, $V > 0$).