Astro 406
Lecture 10
Sept. 18, 2013

Announcements:

- **PS 3 available, due Friday**
  uses real online data: need internet connection
  typo: 1(c) should refer to result from 1(b)
- Office hours: today 1–2pm or by appt
  TA: tomorrow 1–2pm
- ASTR 401: outline due Monday
- Research opportunity: Prof. Ryan Foley `rfoley@cfa.harvard.edu`
  studies exploding stars and dark energy

Last time: mapping Milky Way on 2-D sky and in 3-D space
Q: stellar populations–what? where?

Today:
the dynamical Milky Way: gravity

Local stars: the Solar Neighborhood

Want to determine:
what is the number density of stars?
what is distribution of masses, luminosities?

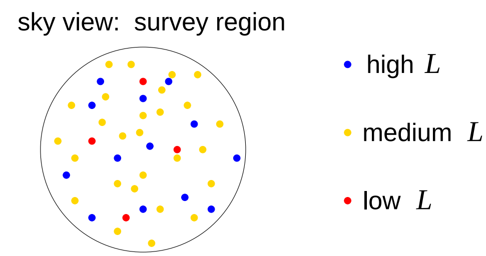
Project: **survey a region of sky**

- measure spectrum, distance to each star
- determine luminosity, mass

In the real world:
any telescope has finite collecting area
and finite sensitivity in photodetectors
$\rightarrow$ only detects stars with brightness above *flux limit* $F_{\rm min}$

iClicker Survey: Bias in Flux-Limited Surveys

consider a flux-limited survey
of stars having a wide range of luminosities



survey finds *more* high-L than low-L stars

From this information alone, what can we conclude?

A high-L stars have a higher density = are more common

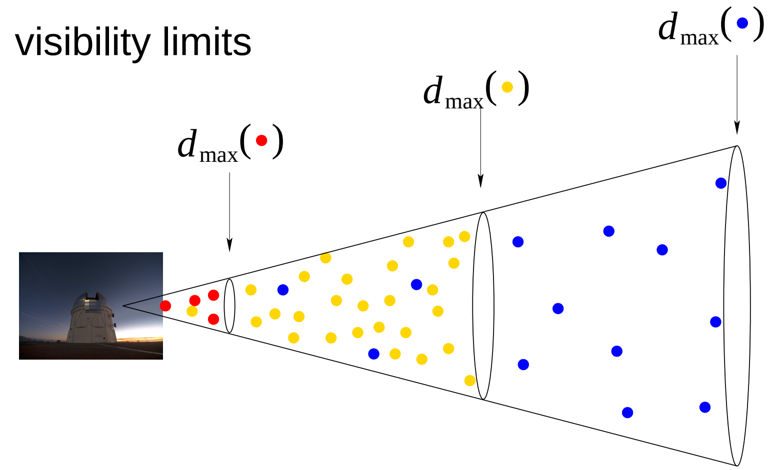
B high-L stars can be seen at a much greater distance but are actually *less* common than low-L stars

C stars of all L are equally common

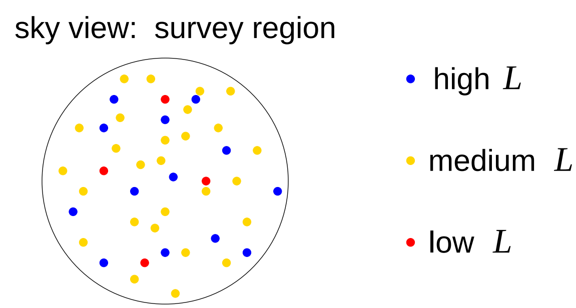
D not enough information given

Flux-Limited Surveys

stars of luminosity L can be seen
in a survey of flux limit F_{min}
out to distances
$d \leq d_{\text{max}} = \sqrt{L/4\pi F_{\text{min}}}$



Observed star counts at different L
reflect both star densities
but also large differences in d_{max}



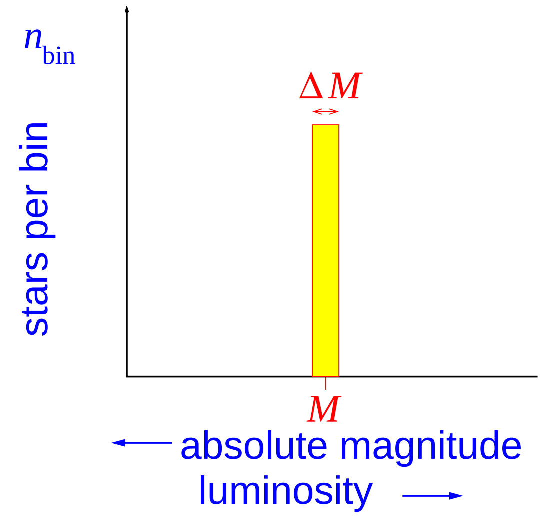
“Malmquist bias” – different *observable* volumes $V_{\text{obs}}(M)$
for different L (or abs mag M)
Q: how to correct for this bias?

We can correct for bias:

in absolute magnitude "bin"

of range ΔM centered on M

- count stars: $N(M, \Delta M)$
- compute observable volume $V_{\text{obs}}(M)$
  vol in which lum $L(M)$ gives flux $\geq F_{\text{min}}$



then: number density ($\equiv$ # stars per unit volume)

in "bin" $M \pm \Delta M/2$ is

$$n_{\text{bin}} = \Phi(M)\ \Delta M \equiv \frac{N(M, \Delta M)}{V_{\text{obs}}(M)} \tag{1}$$

defines **luminosity function** $\Phi(M)$

"bar graph" (= distribution) of star counts

at each absolute magnitude M (i.e., luminosity)

iClicker Poll: Star Demographics

Luminosity function: star number density at each L

What will it look like?

A peaked at *low* L: most stars low mass & wattage

B peaked at *high* L: most stars high mass & wattage

C peaked at *middle* L: most stars medium mass & wattage

D none of the above

`www: luminosity function`

Q: where is it peaked? what does this mean?

Q: possible worries about remaining bias?

The Disk Star Luminosity Function

observed luminosity function for Galactic disk stars:

star *number* vs L

- highly peaked at *low* L and thus *low mass*
  also low temperature: main sequence M, L, T dwarfs
- low mass stars are, by far, the most abundant

star *mass* vs L

- *broad peak* spanning *low/intermediate* L*, mass*
- wide range of stars contribute to Galaxy's star mass

star *luminosity* vs L

- strongly peaked at *high* L*, high mass*
- most massive, luminous stars dominate Galaxy's light output

Star Properties: Local Solar Neighborhood

Sum over all star luminosities to get totals

- **number density** $n_{\rm tot} \approx 0.055$ stars pc^{-3}
- **luminosity density** $\mathcal{L}_{\rm tot} \approx 0.038\ L_\odot$ pc^{-3}, $\sim 75\%$ from MS stars
- **mass density** $\rho_{\rm tot} \approx 0.025\ M_\odot$ pc^{-3}

Average properties:
mean star luminosity $L_{\rm avg} = \mathcal{L}_{\rm tot}/n_{\rm tot} = 0.7 L_\odot$
mean star mass $m_{\rm avg} = 0.45 M_\odot$

∞

Star Properties: Milky Way

extrapolate to entire Galaxy:

- total gas mass $M_{\rm gas} \sim 10^{10} M_\odot$
- total stellar mass $M_\star \sim 10^{11} M_\odot$

  and to the number of Milky Way stars is roughly

  $\mathcal{N}_\star = M_\star / m_{\rm avg} \sim 2 \times 10^{11} \simeq 200$ billion stars

thus: today, $M_\star \gg M_{\rm gas}$

Q: implications of $M_\star \gg M_{\rm gas}$?

But all this mass gravitates!

Q: consequences individual stars, parcels of gas?

Q: consequences for global distribution of stars, gas?

The Dynamical Galaxy

stars, gas have mass $\rightarrow$ gravity
everything pulls on everything else
$\rightarrow$ individual stars and gas parcels
feel net gravity of rest of Galaxy
$\rightarrow$ in general, this is not zero!

but nonzero gravity $\rightarrow$ force $\rightarrow$ acceleration
$\rightarrow$ motion!

In general: everything in Galaxy moves
and has always moved!
$\rightarrow$ present (and past) Galactic structure
both causes and responds to orbits

to understand this interplay
$\rightarrow$ need gravitation technology

Newtonian Gravitation

Strategy:

- we know: gravity of point mass
- we want: generalize to gravity from mass distribution

Q: what is gravity of point mass?

Q: how describe (quantify) a mass distribution in space?

To generalize:

Q: what if two point masses?

Q: what if two N point masses?

Q: what if continuous mass distribution?

Gravity of a Point Mass

consider point mass M at separation $\vec{r}$
grav. force on "test mass" m:

$$\vec{F}_m = -\frac{GMm}{r^2}\hat{r} \tag{2}$$

inverse square *Q: why minus sign?*
$\hat{r} = \vec{r}/r$: unit vector along $\vec{r}$

equation of motion for m:

$$m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2} \equiv m\ddot{\vec{r}} = \vec{F}_m$$

$$\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r} = -\frac{GM\vec{r}}{r^3}$$

acceleration independent of m
Q: physically, what does this mean?
"equivalence principle"

Newtonian Gravitational Field

for point mass M:

- acceleration independent of test mass
- thus only depends on "source" M

formally: can write test mass force $\vec{F}_m = m\vec{g}$
and thus in the presence of a gravity source M
i.e., given the existence and amount *mass*
any and all test particles at point $\vec{r}$ feel acceleration

$$\vec{a} = \vec{g}(\vec{r}) \tag{3}$$

$\Rightarrow$ physical interpretation: each mass M sets up
its own **gravitational field $\vec{g}$** throughout space

Gravity from many sources: Superposition

Thus far: only considered single point masses
what if we add more gravity sources–i.e., more masses?

If 1 point particle m_1 at $\vec{r}_1$
gravity is

$$\vec{g}_1 = -\frac{Gm_1}{r_1^2}\hat{r}_1 = -\frac{Gm_1\vec{r}_1}{r_1^3} \tag{4}$$

For 2 point particles m_1, m_2 at r_1, r_2
gravity is **superposition** of vectors

$$\vec{g} = \vec{g}_1 + \vec{g}_2 = -G\left[\frac{m_1\vec{r_1}}{r_1^3} + \frac{m_2\vec{r_2}}{r_2^3}\right] \tag{5}$$

Q: what's g like between particles? at large distances?

multiple point particles $m_1, ..., m_N$, $\vec{r}_1, ..., \vec{r}_N$:
$\vec{g}$ from superposition:

$$\vec{g} = \sum_{i=1}^{N} \vec{g}_i \qquad (6)$$

$$= -G \sum \frac{m_i \vec{r}_i}{r_i^3} \qquad (7)$$

in general: complicated!
e.g., in Milky Way, sum includes 200 billion stars

for continuous mass distribution, i.e., smooth mass density:
each mass element $dm = \rho \; dV$
at position $\vec{r}$, sum field contribution
$d\vec{g} = -G \; dm \; \vec{r}/r^3$

$$\vec{g}(\vec{r}) = -G \int_V dm \frac{\vec{r}}{r^3} = -G \int_V d^3\vec{x} \frac{\rho(\vec{r} - \vec{x})}{|\vec{r} - \vec{x}|^3} \qquad (8)$$

“highly nontrivial” !

there has to be a better way!

Gauss' Law for Gravity

how sum up? how do the integral?

You already have the technology! Notice similarity:

	Electrostatics	*Gravity*
"charge"	q	m
force	$qQ/4\pi\epsilon_0 r^2\ \hat{r}$	$-GmM/r^2\ \hat{r}$
field	$\vec{F}_q = q\vec{E}$	$\vec{F}_m = m\vec{g}$

formally identical inverse square law forces!
(except sign, and $\pm q$ allowed, $m \geq 0$)

So: can import electrostatics technology
Memory lane: Gauss' Law from EM
`www: PHYS 212`

Gauss' Law in E&M

consider a point charge Q

enclose in sphere: $\vec{E}$ normal to surface $\vec{S}$

$$\int_S \vec{E} \cdot d\vec{S} = E \int_S dS = \frac{Q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0} \tag{9}$$

miracle: holds for all $\vec{E}$ and surfaces $\vec{S}$

$$\text{electric flux} = \int_S \vec{E} \cdot d\vec{S} = \frac{q_{\rm enc}}{\epsilon_0} \tag{10}$$

where $q_{\rm enc}$ is total charge enclosed in surface S

Gauss' Law for gravity: for point mass M

$$\int_S \vec{g} \cdot d\vec{S} = -\frac{GM}{r^2} 4\pi r^2 = -4\pi GM \tag{11}$$

and in general:

$\int_S \vec{g} \cdot d\vec{S} = -4\pi G M_{\rm enc}$

Gauss' Law Example

spherical mass distribution $\rho(r)$

rotational symmetry:

- $\vec{g}$ direction radial: $\widehat{r}$
- $\vec{g}(r, \theta, \phi) = \vec{g}(r)$

Gauss' Law: choose spherical surface

$$\int_S \vec{g} \cdot d\vec{S} = 4\pi r^2 g(r) = -4\pi G m(r) \quad (12)$$

where $m(r) = 4\pi \int dr r^2 \rho(r)$

solve:

$$\vec{g} = -\frac{Gm(r)}{r^2}\hat{r} \tag{13}$$

note similarity to point-source formula
but this works for *any* spherical mass distribution
and works inside, outside mass distribution!

Q: field at center?
Q: field if hollow out inside and you're there?
$\Rightarrow$ field is same as if interior mass concentrated at center!

The Care and Feeding of Gauss' Law

Gauss for gravity:

$$\int_S \vec{g} \cdot d\vec{S} = -4\pi G M_{\rm enc} \tag{14}$$

note: holds for *any* surface S

$\rightarrow$ *you get to choose* S

a powerful tool *if* good choice of surface S

- need to guarantee that $\vec{S} \| \vec{g}$
- need to pick surface where $g = |\vec{g}|$ is constant

then: $\int_S \vec{g} \cdot d\vec{S} = gS$

and can easily solve for $g = -4\pi M_{\rm enc}/S$

How do you make these good choices?

$\rightarrow$ use symmetry of system

Director's Cut Extras

each bin contains some number of stars
with fixed (known) mass, luminosity

to find totals, just sum over each bin

total *number density* of all stars is

$$n_{\text{tot}} = \sum_{\text{bin}} n_{\text{bin}} = \sum_{\text{bin}} \Phi(M) \Delta M \tag{15}$$

total *luminosity density*: weight bins with luminosity

$$\mathcal{L}_{\text{tot}} = \sum_{\text{bin}} L(M)\ \Phi(M) \Delta M \tag{16}$$

total *mass density*: weight bins with mass

$$\rho_{\text{tot}} = \sum_{\text{bin}} \mathcal{M}(M)\ \Phi(M) \Delta M \tag{17}$$