Astro 406
Lecture 18
Oct. 7, 2013

Announcements:

- **PS 6 due Friday**
- ASTR 401: draft section due today

Guest Cosmologist: Prof. Charles Gammie returns
famed for Sgr A* weather forecast
and for building black hole death rays

black hole accretion simulation

Last time: orbits in globular star clusters

for a perfectly spherical mass density

Q: what conservation laws apply to each star?

Q: what do typical orbits look like?

`www: awesome orbit simulator`

Q: why does the cluster have a spherical shape?

Q: how would you guess it got this way?

Globular Cluster Dynamics: Collisions

consider a single star somewhere in the cluster

nearest neighbor distance

$$\ell_{\mathrm{nearest}} \sim n_\star^{-1/3} = \left(\frac{\rho}{m_\star}\right)^{-1/3} \sim 0.1 \text{ pc} = 20,000 \text{ AU} \qquad (1)$$

yikes!

raises issues we must sort out:
which dominates the gravity on a typical cluster star:
• gravity due to very nearest few neighbors, or
• gravity due to numerous but much more distant cluster stars?
Q: influence of nearest neighbor? next-to-nearest? etc?
Q: how could you quantitatively estimate important effects?
  e.g., important distance scales, time scales?

A Comparison of Timescales

orbit timescale around nearest star, for $m \sim 1 M_\odot$

$$P_{\text{orbit}} \sim \sqrt{\frac{\ell_{\text{nearest}}^3}{Gm}} \sim 1/\sqrt{G\rho} = \tau_{\text{grav}} \sim 10^6 \text{ yr} \quad (2)$$

crossing times

to go a distance ℓ_{nearest} with speed $v \sim 5$ km/s

$$t_{\text{cross,neighbor}} \sim \frac{\ell_{\text{nearest}}}{v} \sim 2 \times 10^4 \text{ yr} \quad (3)$$

to go across the whole cluster

$$t_{\text{cross,cluster}} \sim \frac{R_{\text{cluster}}}{v} \sim 10^6 \text{ yr} \quad (4)$$

note the (apparent) miracle: $t_{\text{cross,cluster}} \approx \tau_{\text{grav}}$!

Q: why?

Hint: recall significance of τ_{grav} in Kepler problem,
and in Sun's vertical oscillations in disk

mean free time between "contact" collisions
i.e., collisions in which stars actually touch
...and presumably merge together

if physically touch, collision cross section is
just *geometric cross section* $\sigma_{\rm geom} \approx \pi R_\star^2 \approx \pi R_\odot^2$
and mean free time between merger collisions is

$$\tau_{\rm collide,touch} = \frac{1}{n\sigma_{\rm geom} v} \approx 10^{17} \text{ yr} \tag{5}$$

but HR diagrams showed cluster ages $\sim 10^{10}$ yr *Q: So?*

Q: what about scattering? is this already included $\tau_{\rm collide}$*?*

Gravitational Scattering: Back of the Envelope

expect non-collisional scattering when interacting objects

- exert forces that "reach out" beyond physical size
  e.g., via gravity or electromagnetic effects
- but the objects are *unbound*, e.g., $E_{\rm tot} > 0$

for test mass m in gravity field of point mass M
unbound $E > 0$ orbits are hyperbolae
characteristic lengthscale is roughly set by (PS6)
location $r_{\rm grav}$ where $|E_{\rm grav}| = E_{\rm tot} = m v_\infty^2/2$:

$$r_{\rm grav} = \frac{GM}{v_\infty^2} \approx 10^4 R_\star \approx 10^{-3} \ell_{\rm nearest} \tag{6}$$

using this "zone of gravitational influence"
with cross section $\sigma_{\rm scatter} = \pi r_{\rm grav}^2$

$$\ell_{\rm mfp,scatter} = \frac{1}{n \sigma_{\rm scatter}} \sim 2 \times 10^3 \ {\rm pc} \tag{7}$$

$$\tau_{\rm scatter} = 5 \times 10^8 \ {\rm yr} \tag{8}$$

To summarize timescales:

$$
\begin{aligned}
P_{\text{orbit}} &\sim 10^6 \text{ yr} \sim \tau_{\text{grav}} \sim t_{\text{cross,cluster}} && (9)\\
t_{\text{cross,neighbor}} &\sim 2 \times 10^4 \text{ yr} && (10)\\
\tau_{\text{contact}} &\sim 10^{17} \text{ yr} && (11)\\
\tau_{\text{scatter}} &\sim 5 \times 10^8 \text{ yr} && (12)\\
t_{\text{age,cluster}} &\sim 10^{10} \text{ yr} && (13)
\end{aligned}
$$

Given these:

Q: what kinds of orbits would you expect?

Q: role of collisions? scattering? other gravity effects?

Q: what gravity field does a star typically see?

We see a range of very different timescales

i.e., a *hierarchy* of timescales

- $t_{\text{cross,neighbor}} \ll P_{\text{orbit}}$: cross too quickly for strong perturbation by nearest neighbor (i.e., $v \gg v_{\text{esc,nearest}\star}$)
- $\tau_{\text{contact}} \gg t_{\text{age,cluster}}$: *"touching" collisions never happen!*
- $t_{\text{cross}} \ll \tau_{\text{scatter}}$: *scatterings rare*—not important for most orbits
- $\tau_{\text{scatter}} \ll t_{\text{age,cluster}}$: scatterings do eventually happen

Bottom line:

- rarely influenced by (scattered by) nearest neighbors on vast majority of orbits, don't "notice" them
- but *do* feel net gravity of all stars
  $\rightarrow$ see spatially "smooth" gravity field
- scatterings *do* happen eventually
  these redistribute energy, lead to equilibrium state!

Q: a familiar system of many scattering particles?

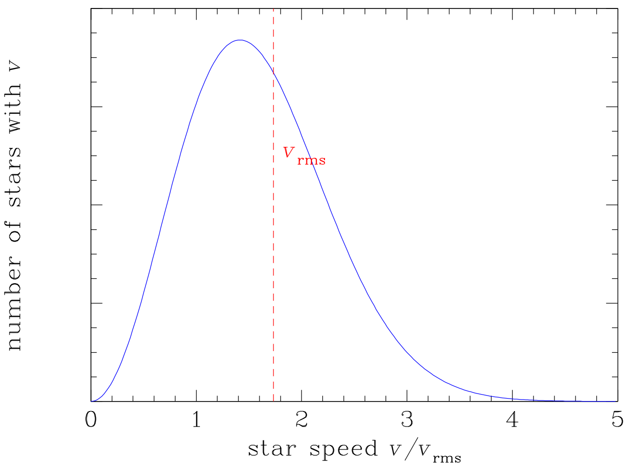
Q: what is its equilibrium state?

GC Temperature and the Virial Theorem

bad news:
• 10^6 orbits complex $\rightarrow$ can't solve 'em all

good news:
• don't really want a million orbits anyway
$\Rightarrow$ *can* find **average** properties and
distributions of positions, velocities

GC are old: reached *equilibrium*
$\rightarrow$ like a gas coming into thermal equilibrium
GC "gas" of stars has thermal v distribution
(i.e., a Maxwellian)



Q: if thermal, how are T,v connected?

In thermal equilibrium:

the *average* KE of *each particle* (here: star)

$$\frac{1}{2}\, m v_{\rm avg}^2 = const = \frac{3}{2}\, kT \tag{14}$$

and so average (root-mean-square) speed:

$$v_{\rm rms} = \sqrt{\frac{3kT}{m}} \tag{15}$$

but have already seen density distribution with
speed distribution constant with r
can describe as isothermal sphere! (PS2)

But wait, there's more!
GC size, speeds ($\rightarrow T$) related by
Virial Theorem

Virial Theorem
Very important result:

$$2\langle KE\rangle + \langle PE\rangle = 0$$

key assumptions:
(1) gravitating system, (2) no other interactions

Apply to a globular cluster: system of $N \approx 10^6$ stars

Roughly: for N stars
$2KE \sim \ Nmv^2$ *Q: why?*
$PE \sim -GN^2m^2/2R$ *Q: why?*
$\rightarrow$ if in gravitational equilibrium, then
$v_{\rm vir}^2 \sim GNm/2R = GM/2R$

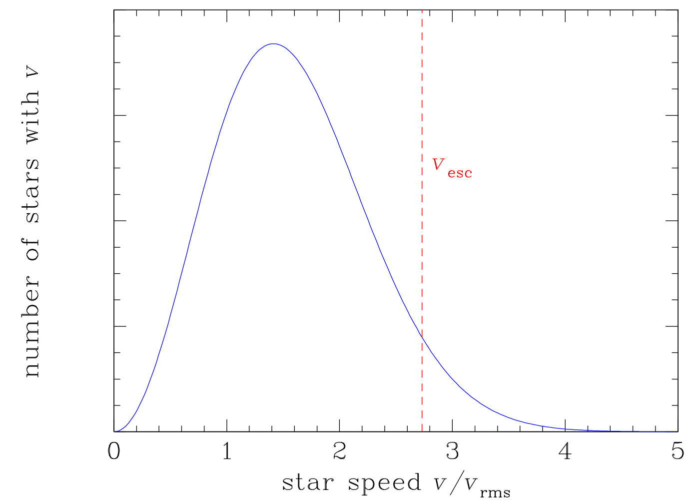
observed globular clusters: $v_{\rm obs} \simeq v_{\rm vir}$!

- behold the power of the virial!
- confirms equilibrium nature of star motions

Globular Cluster Evaporation

GC star velocities change due to:
- motion in global GC potential
- scattering off individual stars (rare)

if ever $v > v_{\rm esc} \approx \sqrt{2GM/R}$
star escapes!



recall: in static potential Φ
star energy $E = 1/2\ mv^2 + m\Phi$ conserved
$\rightarrow$ won't escape if haven't yet
Q: why does this follow?

Q: but what could lead a star to escape?