

Astro 406  
Lecture 33  
Nov. 13, 2013

Announcements:

- **PS 10 due Friday**
- ASTR 401: make appointment to meet
- Office Hours: today 1–2pm or by appt  
TA Office Hours: tomorrow 1–2 pm

Last time: cosmic expansion history measured

*Q: results?*

*Q: implications? simplest explanation?*

# An Accelerating Universe: Implications

SN Ia:  $\ddot{a} > 0$

Friedmann:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G \left( \rho + 3\frac{P}{c^2} \right) \quad (1)$$

$$\Rightarrow \rho + 3P/c^2 < 0$$

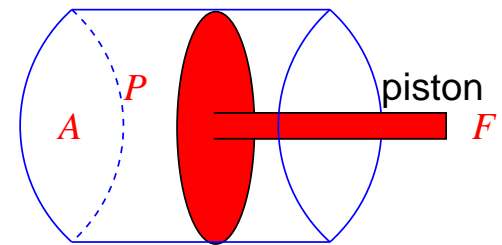
$$\Rightarrow P < -\rho c^2/3 \text{ negative pressure!?!}$$

Physical “interpretation”:

recall:  $F = \int P dA$

$P > 0$ : outward force (e.g., ideal gas)

$P < 0$ : inward force (e.g., elastic)



# Cosmic Acceleration: Simplest Solution

revive Einstein “**cosmological constant**”

introduce *new constant of nature*  $\Lambda$

with  $\Lambda$ , Friedmann becomes:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R_0^2 a^2} + \frac{\Lambda c^2}{3} \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho + 3\frac{P}{c^2}\right) + \frac{\Lambda c^2}{3} \quad (3)$$

notice the nice features:

- with  $\Lambda$ ,  $H$  changes more slowly than without
- $\Lambda$  term positive in acceleration equation

$\omega \rightarrow$  naturally leads to acceleration

# Cosmological Constant and Newtonian Gravity

the cosmo constant is not just for cosmology!  
once the  $\Lambda$  genie is out of the bottle  
also affects Newtonian gravity

at distance  $\vec{r}$  from a mass  $M$ , *Newtonian acceleration becomes*

$$\vec{g} = -\frac{GM}{r^2}\hat{r} + \frac{\Lambda c^2}{3}\vec{r} \quad (4)$$

Q: *g* behavior at small  $r$ ? large  $r$

Q: behavior as  $M \rightarrow 0$ ?

Q: particle motions in this gravity field?

⊢

Q: in simple terms, what does the cosmo constant do?

## An Accelerating Universe: Implications

Recall: expected **d**eceleration because ordinary matter  
(even dark matter!) has gravitational **a**ttraction  
matter-filled universe should have **s**lowing expansion  
→ if matter is all there is, U should **d**ecelerate

But: found **a**cceleration – exact opposite of intuition  
→ something present which has gravitational **r**epulsion!  
→ Universe seems to contain something having **“antigravity” !?!**  
...and huge amounts of such stuff!  
enough of it to overwhelm the attraction of ordinary matter!

Q: so what is the fundamental difference between dark matter  
and dark energy?

# The Dark Sector

## dark matter

motivated by galaxy rotation curves

*invented to prevent galaxies from flinging apart*

adds extra mass → galaxies have more *attraction*

## dark energy

motivated by cosmic acceleration

*invented to make the universe fling apart*

adds negative pressure → cosmic *repulsion*

we can choose to encode  $\Lambda$  by an effective density and pressure:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho_{\text{tot}} - \frac{kc^2}{R_0^2 a^2} \quad (5)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho_{\text{tot}} + 3\frac{P_{\text{tot}}}{c^2}\right) \quad (6)$$

where  $\rho_{\text{tot}}$  includes  $\rho_\Lambda = \Lambda c^2/8\pi G$  “vacuum energy density”

$P_{\text{tot}}$  includes  $P_\Lambda = -\rho_\Lambda c^2$  “vacuum pressure” www: SN cosmo results

SN Ia:  $\Omega_\Lambda \simeq 0.7$ ,  $\Omega_m \simeq 0.3$

→ independent evidence for  $\Omega_{\text{not matter}} \simeq 0.7!$

www: cosmic pie chart

## $\Lambda$ , Geometry, and Fate

Note: if  $\Lambda \neq 0$ , then  
geometry  $\Leftrightarrow$  fate no longer linked!

That is, can mix-n-match curvature and fate

In particular,

if  $\Omega_{\text{matter}} = 0.3$ , and  $\Omega_{\Lambda} = 0.7$

**universe expands forever**

in fact, for  $t \gg t_0$ ,

$$a(t) \rightarrow e^{+\sqrt{\lambda/3}ct}$$

*Q: which means?*



## Living with $\Lambda$

if cosmic acceleration is due to  $\Lambda$ :  $a(t) \rightarrow e^{+\sqrt{\Lambda/3}ct}$   
expand forever *exponentially*

**The Future:**  $a \gg 1$

- as,  $\rho_{\Lambda}/\rho_{\text{matter}} \propto a^3 \gg 1$ : matter becomes unimportant
- $H^2 \rightarrow \Lambda c^2/3$ : Hubble becomes a true constant!

**The Past:**  $a \ll 1$

- $\rho_{\Lambda}/\rho_{\text{matter}} \propto a^3 \ll 1$  in early U
- cosmo constant ignorable

www: cosmic coincidences

- ◊ Raises question: *Why do we live just when  $\rho_{\Lambda} \approx \rho_{\text{matter}}$ ?*  
*Q: possible solutions?*

## Dynamic Solution: Dark Energy

invent new substance: **dark energy**

with *density*  $\rho$ , *pressure*  $P$

related by “equation of state”

$$P = w \rho c^2 \quad (7)$$

Friedmann: given  $w$ , can find  $\rho(a)$

in fact,  $\rho \propto a^{-3(1+w)}$

SN Ia:  $w < -0.7$ , i.e.,  $P < 0$ !

www: current data on  $w$

Cosmological constant has  $P_\Lambda = -\rho_\Lambda c^2$

and thus  $w_\Lambda = -1$  exactly and forever

→  $\Lambda$  is a special case of dark energy

→ dark energy generalizes the cosmo constant

dark energy:  $P = w\rho c^2$

with  $w < -0.7$ : strongly negative pressure

what substance acts this way?

elementary particle physics:

if vacuum has energy (quantum fluctuations) can get  $P < 0$

cosmo constant  $\Lambda$  equivalent to  $w = -1$

but more generally, could have evolving  $w = w(z)$  “quintessence”

can find models where DE evolves to keep  $\Omega_{\text{DE}} \sim \Omega_{\text{matter}}$

→ fixes the coincidence problem of why this is true now

# Phantom Energy

dark energy “equation of state”  $P = w\rho c^2$

current data: consistent with  $\Lambda$

- best-fit  $w \approx -1$
- no strong evidence for  $w$  change with time or redshift

...but  $w < -1$  not ruled out (“phantom energy”)

www: current data on  $w$

What if  $w < -1$ ?

future cosmic evolution: after time  $\Delta t = t - t_0$ , scale factor is

$$a(t) = \left( \frac{t_r}{t_r - \Delta t} \right)^{\frac{2}{3|w+1|}} \quad (8)$$

where  $t_r \sim 11 |w + 1|^{-1}$  Gyr *is a constant*

Q: what does this mean for the future?

## Dark Energy Doomsday?

The “phantom energy”  $w < -1$  future:  
after time  $\Delta t$ , scale factor is

$$a(t) = \left( \frac{t_r}{t_r - \Delta t} \right)^{\frac{2}{3|w+1|}} \quad (9)$$

has  $a \rightarrow \infty$  when  $\Delta t = t_r$ !

i.e., *infinite expansion* occurs a *finite time* from now!

*Q: how does  $a(t)$  look for this universe?*

it gets worse...

consider dark energy density  $\rho_{\text{DE}} \propto a^{-3(1+w)}$

*Q: how does this evolve if  $w < -1$ ?*

*Q: implications?*

## The Big Rip

dark energy density  $\rho_{\text{DE}} \propto a^{-3(1+w)}$

if  $w < -1$ , then  $\rho_{\text{DE}}$  *grows* as  $a$  grows

→ expansion increases phantom energy density!

→ expand more → drive up  $H$  further!

as  $t_r$  approaches, and  $a \gg 1$

dark energy density inside bound structures  
eventually overwhelms their binding energy

result: *all bound structures torn apart*

clusters, galaxies, planets, people, atoms...

→ all particles separated from all others

“cosmic doomsday” → **Big Rip**

Cosmologists & Ghostbusters Harold Ramis & Bill Murray  
Drs. Peter Venkman & Egon Spengler

Dr. Spengler: It would be bad.

Dr. Venkman: I'm fuzzy on the whole  
"good/bad" thing here. What do you mean "bad"?

Dr. Spengler: Try to imagine all life as you know it  
stopping instantaneously and every molecule  
in your body exploding at the speed of light.

Dr. Venkman: Right. That's bad. Okay. All right.  
Important safety tip.

*Cosmologist David Letterman reacts*

From the Wahoo Gazette, July 23, 2003

Dave was enjoying a nice read of the New York Times after Tuesday's show when he came upon an article [on page 19] which claims the universe is splitting in two. Some "dark energy" is wrenching the universe apart.

Dave says "If the world and the universe is actually splitting apart like the New York Times says, then **WHY IS IT ON PAGE 19?** Shouldn't it be on Page 1?" For the rest of the program, everything seems minor and inconsequential considering that the universe is being pulled apart by a mysterious dark energy.



# Living with Dark Energy

## Current Data:

acceleration demands dark energy of some kind

cosmo constant is simplest version, but also works well

→ no current data demand something more complicated than  $\Lambda$

→ but other dark energy models survive

→ big rip *not required* or strongly favored, but also

*can't be ruled out* by current data!

## Outlook:

most of universe dominated by a force/substance

we don't at all understand!

- major cosmo puzzle!

- upcoming projects to better measure " $\Lambda$ ,"

or whatever dark energy is...

or to uncover mistake! could this be our "ether?"

★ either way Illinois a major player in making progress!

## iClicker Poll: The Reason for Cosmic Acceleration

Vote your conscience!

Of these basic explanations for cosmic acceleration

Which do you think is right?

- A** General Relativity *correct*, and there is a cosmological constant  $\Lambda$
- B** General Relativity *correct*, and  $\Lambda = 0$ , but the Universe contains evolving dark energy
- C** General Relativity *incorrect*, and the Universe only contains matter

## Revolution Re-Re-Re-Visited

### **Copernican Revolution I (17th Century):**

Earth is one typical planet among many  
not center of solar system

### **Copernican Revolution II (earth 20th Century):**

Sun is one typical star among many  
not center of Milky Way Galaxy

### **Copernican Revolution III (1920's):**

Milky Way is one typical galaxy among many  
Universe much larger than previously thought

### **Copernican Revolution IV (late 20th century):**

most matter in the U is weakly interacting dark matter  
we are not even made of the dominant stuff

### **Copernican Revolution V (21st century):**

most of energy content of U is dark energy  
most of the U isn't made of matter at all!

... stay tuned for more...