Astro 210
Lecture 5
Sept. 6, 2013

Announcements:

- **PS 1 due now**
- **PS 2 available, due next Friday Sept 13**

Last time:

▷ star distances

Q: what's the "gold plated" way to measure distance?

update: 2013 `www: discovery` of closest star system in a century

Q: why can't we use this technique for all cosmic objects?

- a stellar census: L and T – the Hertzsprung-Russell diagram

Q: what does the HR diagram look like for a "fair sample" of stars?

Q: main features? where does the Sun fit in?

Weighing Stars

We saw that clever measurements give a stars
- luminosity
- surface temperature
- radius

What about mass?

For single stars:
mass determination difficult, very indirect
but we *can* find masses for stars in **binary** systems
see Director's Cut Extras bonus tracks

iClicker Poll: Star Masses

Vote your conscience!

Measure *mass* M and and *luminosity* L for main sequence stars
plot L vs M; each star is one (M, L) point
What trend(s) will we find?

A M and L tightly related: L increases with M
(more massive = more luminous)

B M and L tightly related: L decreases with M
(more massive = less luminous)

C M and L unrelated: large spread in L for each M

D none of the above

Q: what does this mean for H-R diagram?

Star Masses

For main sequence stars:
data show very tight trend: $\log L = \alpha \log M + const$, $\alpha \approx 3.5$
$\rightarrow$ solve to find $L \propto M^{\alpha}$, and use $L(M_\odot) = L_\odot$:

$$L = \left(\frac{M}{M_\odot}\right)^{\alpha} L_\odot \approx \left(\frac{M}{M_\odot}\right)^{3.5} L_\odot \tag{1}$$

very strong increase of L with M!
example: $L(2M_\odot) = 2^{3.5} L_\odot = 11 L_\odot$

Q: which has more total lum?
2 stars at $1M_\odot$, or 1 at $2M_\odot$?

☞ Lesson for H-R diagram: main sequence is really
a sequence in mass

Star Lifetimes

we saw: stars have life cycles, live for finite time
lifespan τ set by *energy conservation*

total energy *emitted* over star's life

$$E_{\text{emitted}} = \int_0^{\tau} L(t)dt \approx L\tau \tag{2}$$

must come from star's fuel supply: $E_{\text{emitted}} = E_{\text{fuel}}$ (cons of E)
so fuel = mass × burn rate

but star's fuel comes from their mass: fuel $\propto M$

$$\tau \approx \frac{E}{L} \propto \frac{M}{L} \propto M^{1-\alpha} = M^{-2.5} \tag{3}$$

high $M \rightarrow$ high $L \rightarrow$ short τ
→ on H-R: main sequence is also sequence of lifespans

Lesson: **mass is most important parameter of a star!**

a star's mass at birth (mostly) determines its future life cycle

other factors: composition, rotation, binary companion

imagine a large number of stars are born at the same time
with a range of masses spanning high to low

Q: what will happen as time goes on?

Q: how will will the system's HR diagram change?

Q: how will the system's color change?

`www: spiral vs elliptical galaxies`

Q: implications?

Stellar Evolution: Hydrostatic Equilibrium

the life of each and every star is a battle:
a *constant struggle against its own gravity*

inward gravity force $GM_{\text{interior}}m/r^2$ must be counterbalanced
or the star will collapse under its own weight

last Friday we saw: in Sun and main sequence stars
pressure provides the opposing force

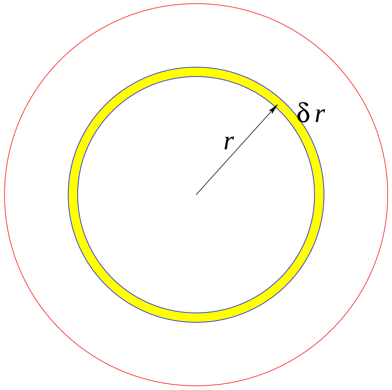
Consider a shell of gas in the Sun, **at rest**

radius r, thickness $\delta r \ll r$

shell area $A = 4\pi r^2$

shell volume

$$V = \frac{4\pi}{3}[(r+\delta r)^3 - r^3] \approx 4\pi r^2\, \delta r = A\, \delta r$$



shell mass $m_{\text{shell}} = \rho V = \rho A\ \delta r$
shell weight $F_{\text{W}} = -g m_{\text{shell}} = -g\rho A\ \delta r$

upward force

pressure: on bottom $P(r)$, on top $P(r+\delta r)$

net upward force

$$F_{\mathrm{p}} \;=\; \Delta P \times A = [P(r+\delta r) - P(r)]A = A\,\frac{dP}{dr}\,\delta r$$

hydrostatic equilibrium: $F_{\mathrm{weight}} = F_{\mathrm{pressure}}$

upward pressure exactly balances downward gravity

$\Rightarrow dP/dr = -g\rho = -GM(r)\rho(r)/r^2$

Note what this means:

$\rightarrow$ Sun's mechanical structure $\rho(r), M(r)$ intimately related to thermal structure $P(r) = \rho kT/\mu \propto T(r)$

∞

analogy: balloon, basketball (inward elastic force vs outward P)

Matter: Gasses, Pressure, and Temperature

Take microscopic view of gas: what's going on with atoms?
in any gas (stars, Universe, this room):

- atoms widely spread → empty space between
- *constantly in motion* as free bodies until collision with other gas particles, container walls (if any)
- collisions "scramble"/randomize motion direction and tend to equalize particle energies

on macroscopic scales (i.e., how we see things)
particle motions perceived as *temperature*

Now zoom back to our macroscopic view:

- enclosed gas exerts force–pressure–on walls
  Q: how does atom picture explain this?
- *Q: how does gas change if turn up T? what are atoms doing?*

particle motion → particle kinetic energy
proportional to bulk temperature

$$\langle KE \rangle_{\text{per particle}} = \frac{1}{2}\mu\langle v^2 \rangle = \frac{3}{2}kT \qquad (4)$$

where μ = mass of 1 gas particle: "molecular weight"
and $k = 1.38 \times 10^{-23}$ Joules/Kelvin: Boltzmann's constant

example of general rule of thumb:
in thermal system, typical particle energy $\boxed{E_{\text{particle}} \sim kT}$

for thermal gas: average particle speed ("root mean square") is

$$v_{\text{rms}} = \sqrt{\frac{3kT}{\mu}} \qquad (5)$$

hotter ↔ *faster* particles
colder ↔ *slower* particles

Microscopic View of a Piston

Now consider a large number of gas particles

- in a sealed volume, empty outside
- covered with a *piston* of area A

from microscopic viewpoint:
piston constantly bombarded by gas particles
if let free–would be pushed away

to resist bombardment, must *push* on piston = exert *force* F
define **pressure**

$$P = \frac{\text{force on piston}}{\text{area of piston}} = \frac{F}{A} \tag{6}$$

Q: how to intensify bombardment = pressure on piston?
Hint–more than one way to do this

Pressure

collisions with walls → momentum transfer → force → pressure

`www: piston simulation`

ideal gas

pressure P, volume V, total number N of particles
and absolute (Kelvin) temperature T all related
by *ideal gas equation of state*:

$$PV = NkT \tag{7}$$

- N counts individual particles, typically very large!
  alternatively: can count in units of moles of particles
  i.e., in units of $N_{\text{Avo}} = 6 \times 10^{23}$
  then # moles is $n_{\text{moles}} = N/N_{\text{Avo}}$
  and $PV = n_{\text{moles}} RT$, where $R = N_{\text{Avo}} k = 8.3$ Joules mole^{-1} K^{-1}
- since density $\rho = M/V = \mu N/V$, with μ gas particle mass
  can rewrite ideal gas law: $\boxed{P = \rho kT/\mu}$

Stellar Evolution: Birth and Main Sequence

Stars are born in cold, dense clouds of gas

`www: Eagle nebula`

Q: why is coldness important?

Most of star's life spent on Main Sequence
- in hydrostatic eq.
- "burning" H → He via nuke fusion

Reactions:

$p + p \rightarrow d = \boxed{\text{np}}$ *Q and which other particles?*

$d + p \rightarrow {}^3\text{He} = \boxed{\text{ppn}} + \gamma$

${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} = \boxed{\text{ppnn}} + p + p$

Q: what are particles in first step?

Main Sequence pp Chain

First link: $p + p \rightarrow d = \boxed{\text{np}} + e^{+} + \nu$

d deuterium: *Q: which is what kind of atom?*

e^{+} positron: **antimatter** partner of e^{-}

opposite charge, same mass

ν neutrino: no charge, tiny mass ($m_{\nu} \ll m_{e}$)

very weakly interacting, only created in nuke transformations

neutrinos come directly from solar core → detect on earth

`www:  SNO detector`

`www:  Super-K image of Sun`

⇒ proof Sun powered by fusion!

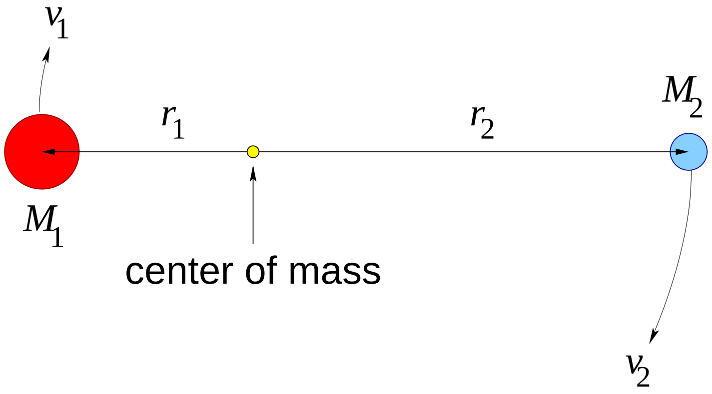
Q: what happens when core of star is all He?

Director’s Cut Extras

Measuring Star Masses: Binary Systems

for single stars without companions: can't accurately find mass

But can find masses for **binary** systems:
two stars orbiting common center of mass



COM positions: $r_1/r_2 = m_2/m_1$

measure P, and r_1, r_2
→ find mass ratio

problem: must measure r's *Q: how?*

Types of Binary Stars

visual binary

can see both stars!

`www: visual binary orbit`

eclipsing binary

stars pass in front of each other

can see in light curve:

diagram: light curve → get rs from timing of eclipses

spectroscopic binary

periodic Doppler shifts in spectrum

see $\Delta\lambda_1$, $\Delta\lambda_2$

→ radial velocity $v_r/c = \Delta\lambda/\lambda_0$

then $v_1 = r_1\omega = 2\pi r_1/P$

can solve for r!