Astronomy 406, Fall 2013
Problem Set #5

Due in class: Friday, Oct. 4; Total Points: 60 + 5 bonus points

1. *A Gut Feeling for Dark Matter.*

   (a) [5 points] Compute the local mass density of dark matter $\rho_{\rm DM}(R_0)$. Assume a spherical dark matter distribution, with a flat rotation curve having $v_0 = 220$ km/s, and our Galactocentric distance $R_0 = 8.5$ kpc. Express your answer in g cm^{-3}.
   Notice that the familiar expression $M = \rho V$ is only true for a spatially constant density. When density varies over space, then in each small volume dV, the mass is $dM = \rho dV$, that is, $\rho = dM/dV$.

   (b) [5 points] The most popular theories for dark matter predict that it takes the form of of weakly interacting massive particles (WIMPs), each of which has mass of roughly $m_{\rm wimp} = 100$ GeV$/c^2$.[1]

      i. Calculate the number density of WIMPs in units of particles per cubic centimeter.

      ii. Then estimate the number of WIMPs inside your body at any given moment, and comment.

2. *Alternatives to Dark Matter.* The dark matter hypothesis provides one explanation for flat rotation curves, but not the only explanation. Another possibility is that we must modify our theory of gravity.

   (a) [5 points] Give at least one reason why we should (at present) remain at least somewhat skeptical about the dark matter hypothesis and thus should keep an open mind about alternatives.

   (b) [5 points] For this problem we adopt the viewpoint that there is *no* dark matter: all of a Galaxy's mass is in the form of ordinary gas, stars, and dust. Instead, we assume that gravitational acceleration g differs from the usual Newtonian behavior.

      i. Consider a galaxy's rotation curve at large distances r. Here, essentially all of the Galaxy's visible (and thus total!) mass M in interior to the orbit.
      Assume gravity provides centripetal acceleration, and use the observed flatness of the rotation curve to trivially solve for $g(r)$.

      ii. Focus on how g depends on r: comment on how your result differs from the usual Newtonian result.

   (c) [5 points] The modified gravity result you found in part (b) must hold only at large distances. At small distances (i.e., Solar System scales) g must revert to the Newtonian behavior.

      i. To see this, assume the contrary. Use the form of $g(r)$ you found in (b) to find a modified version of Kepler's law relating planetary orbit radius a and period P. You should find $P \propto a^n$; don't worry about the proportionalitiy constant, but find the value of n.

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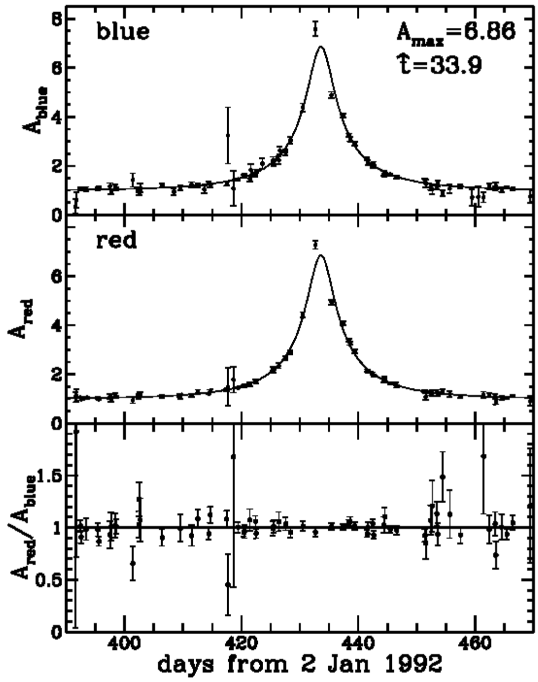
[1] This is a particle physics unit of mass, which makes for easy conversion to rest energy. Since $E_{\rm rest} = mc^2$, we have $m = E/c^2$. For reference, a proton has mass $m_p = 0.938$ GeV$/c^2$, where 1 GeV = 10^9 eV.

ii. Explain how the observed Kepler's third law (i.e., the observed value of n) demands that modified gravity cannot take its large-distance form in the Solar System.

3. *Microlensing and Dark Matter Towards the LMC.* In class it was asserted that the results of the microlensing experiments monitoring the Large Magellanic Cloud (LMC) provide strong constraints on dark matter in the Galactic halo. Here you will flesh out these claims.



(a) [5 points] The first LMC microlensing event detected by the MACHO experiment is plotted in the figure above (Alcock et al. 1993 *Nature* 365, 621). Explain why the bottom panel, plotting $A_{\text{red}}/A_{\text{blue}}$, is important to show, and what conclusions we can infer from it.

(b) [5 points] The microlensing experiments use LMC stars as the background sources; thus, the MACHOs they can detect via lensing are those between us and the LMC. We thus need to know the *total* amount of dark matter (MACHOs or otherwise) between us and the LMC.

Calculate the total Milky Way mass $M(R_{\text{LMC}})$ out to the LMC at distance $R_{\text{LMC}} = 50$ kpc, assuming a constant rotation curve $V(r) = v_c = 220$ km/s. Express your answer in $M_\odot$.

(c) [5 points] The light curves in the top two panels of Figure 1 allow one to calculate the characteristic timescale t of amplification. This is defined to be the time during which $A(t) > 1.34$.

i. Estimate t from Figure 1.

ii. Then use your value and the speed v_c above to compute the Einstein radius $r_{\rm E}$; this should turn out to be ~ 1 AU.

(d) [5 points] You may assume that the lens is halfway between us and the LMC: $D_l = D_{ls} = R_{\rm LMC}/2$, and $D_s = R_{\rm LMC}$.

i. Derive an expression which shows how to find the lens mass m in terms of $r_{\rm E}$.

ii. Using your value for $r_{\rm E}$, calculate m and express your answer in $M_\odot$. You should find $m \sim 0.2 M_\odot$.

(e) [5 points] A more careful analysis of this and subsequent microlensing events shows that the average lens mass is closer to $m \sim 0.5 M_\odot$ (if the events are interpreted as due to MACHOs). The two conventional compact objects with this mass are: (1) a $0.5 M_\odot$ star, and (2) a $0.5 M_\odot$ white dwarf. Explain why a white dwarf is the more likely candidate MACHO.

(f) [5 points] After 5.7 years of running, the MACHO experiment reports a microlensing probability of $P = 1.2 \times 10^{-7}$ per star monitored. That is, the chance of seeing a single star lensed during any particular time is P; the smallness of this number is what demands that so many stars had to be monitored. The probability is related to the mass density $\rho_{\rm macho}$ of MACHOs between us and the LMC, which you may take to be constant.[2] This probability can be computed using the "dart throwing" technology developed in Question 3 of Problem Set 2. Here each line of sight to a LMC star is a dart toss, and each MACHO has a "bullseye" with area $\sigma = \pi r_{\rm E}^2$.
Show that in this picture, the probability of a "hit" (and thus microlensing) is given by

$$P = \frac{\rho_{\rm macho}}{m} \sigma R_{\rm LMC} = \frac{\rho_{\rm macho}}{m} \pi r_{\rm E}^2 R_{\rm LMC} \qquad (1)$$

(g) [5 points] Equation (1) allows you to compute the MACHO mass $M_{\rm macho}(R_{\rm LMC})$ out to the LMC.

i. Do this. Express your answer in $M_\odot$.

ii. How does your answer depend on the assumed lens mass m?

iii. Compare your result for $M_{\rm macho}(R_{\rm LMC})$ with your answer to part (a), and comment.

(h) [5 **bonus** points] If Galactic halo is composed of very old white dwarf MACHOs, these will have several different signatures. Explain one way that evidence of these objects (or material associated with their creation) could in some way be detected observationally. (Your instructor has worked on this problem.)

[2] That is, you may for simplicity ignore the $\rho \propto 1/r^2$ dependence implied by the flat rotation curve.