Astro 406

Lecture 11

Sept. 20, 2013

Announcements:

- **PS 3 due now**
- **PS 4 available; due next Friday**
- iClicker scores posted on Compass; check for accuracy!
- ASTR 401: outline due Monday

Last time: Gravity and galaxy dynamics

- gravity force (weight) test mass $\vec{F} = m\vec{g}$

  Q: is an orbiting astronaut weightless?
- point mass M: $\vec{g} = -GM/r^2\ \hat{r}$
- spherical mass: $\vec{g} = -GM_{\rm enc}(r)/r^2\ \hat{r}$

  Q: Earth surface gravity: hollow core vs dense core?

  Q: in hollow sphere, effect of adding mass outside?

Gravity and Rotation Curves

extremely important special case of dynamics:
circular motion, with only gravity force acting

observable properties:

- distance from center r
- circular speed v_c at r *Q: how to observe?*

$\Rightarrow$ together define **rotation curve**
which as plot of $v_c(r)$ vs r

but gravity determines motion
so speed pattern $\rightarrow$ probes gravity
now can quantify how rotation curves measure gravity

circular motion: centripetal acceleration provided by gravity
$v_{\rm circ}^2/r = g(r)$
where $g(r)$ is gravity acceleration at r

$$v_{\rm circ}(r) = \sqrt{r g(r)} \tag{1}$$

So: **rotation curve measures gravity field $g(r)$!**

For point mass M, then $g(r) = GM/r^2$
$\Rightarrow v_{\rm circ}(r) = \sqrt{GM/r}$ (PS 1)

For spherical mass distribution, $g(r) = Gm(r)/r^2$,
where $m(r) = m_{\rm enc}(r)$ is mass *interior* to or "enclosed" by r
so: rotation curve $\rightarrow$ gravity field $\rightarrow$ mass distribution $m(r)$

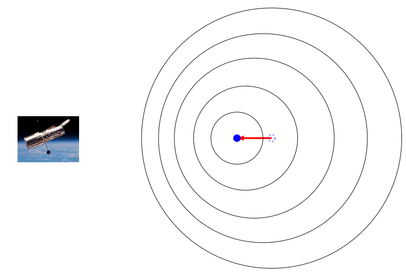
$$m(r) = \frac{r v_{\rm circ}(r)^2}{G} \tag{2}$$

rotation curve "weighs" galaxy!
a powerful tool!

Motions within the Milky Way

measure speeds via *Q: how?*
Doppler effect

measured $\lambda_{\text{obs}} \neq \lambda_0$ rest (lab)
sensitive to **radial=line-of-sight** v component
if $v_r \ll c$:



$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{v_r}{c} \qquad (3)$$

full special relativity result, good for all v_r

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1 \qquad (4)$$

approaching source: $v_r < 0 \Rightarrow \lambda_{\text{obs}} < \lambda_0$: blueshift
receding source: $v_r > 0 \Rightarrow \lambda_{\text{obs}} > \lambda_0$: redshift

Q: what is best way to measure shift for real astronomical objects?

In general, both the Sun and nearby stars all in motion around Galactic center *and* relative to each other

Q: what frame is best to describe our neighborhood?

Local Standard of Rest

the Sun, nearby stars each move w.r.t. the others
average motion of nearby stars:
"local standard of rest"
circular Galactic orbit at our location: $R_0 = 8.5$ kpc

all speeds relative to $\vec{v}_{\rm lsr} = \vec{v}_0$

want to measure both $\vec{v}_0$ and "peculiar" relative motions that deviate from it

If we and all nearby stars moved with local std of rest,
Q: what would nearby star Doppler v_r pattern look like?

Blast From the Past: Circular Motion

Recall that in *circular motion*:

- angular speed $\omega = d\theta/dt = \dot{\theta} = 2\pi/P$
- angular velocity $\vec{\omega}$ *Q: what sets direction?*
- linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$
- (tangential) speed $v = \omega r$, or $\omega = v/r$
- centripetal acceleration $\vec{a}_c = -v^2/r\,\hat{r} = -\omega^2\vec{r}$

Measuring the Milky Way Rotation Curve

want to know circular orbit speed patten $v(R)$
vs Galactocentric radius R
for disk stars, gas

good news:
nature is kind–has given us the Doppler effect
$\rightarrow$ gives speed measurement
and can determine very accurately!

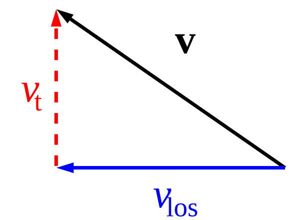
bad news:
Q: what's the catch? or catches?

∞

Relative Motions

Doppler measure star, gas speed that is

- relative to us–and we orbit too!
- velocity component along line of sight v_{los} not transverse component v_{t}



have to

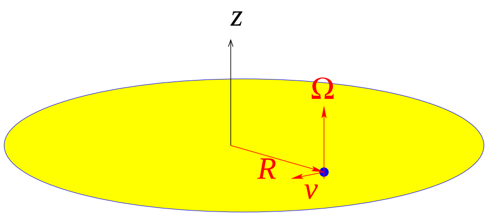
- measure observables $\boxed{v_{\text{los}}}$ scanned across Gal. longitude $\boxed{\ell}$
- work out how to go from these to what desired: $\boxed{v(R)}$

Note: derivation different from SG
same result, but gives another perspective
go with whatever works for you

Characterizing Physical Motions:

in Galactocentric coordinates:

$$\vec{v}(R) = \vec{\Omega}(R) \times \vec{R}$$



and

$$\vec{\Omega}(R) = 2\pi/P(R)\ \hat{z}$$

in magnitude, for circular motion

$$v(R) = \Omega(R)\ R = \frac{2\pi R}{P(R)} \quad (5)$$

so each of $v(R), \Omega(R), P(R)$ encodes equivalent info

The Problem

in general, $v_{\text{los}} \neq v$! some info lost!

To get a feel for what expected, let's try some simple "toy models"

Place Your Bets

Prediction:
imagine $\Omega(R) = const$ vs R for all Galaxy

Q: what is patter of rotation period $P(R)$?

Q: what is MW rotational motion like?

Q: what would $v_{\rm los}$ pattern be?

plot $v_{\rm los}$ vs Gal. longitude ℓ

iClicker Poll: The One Ring

Imagine: *all* gas lies *interior ring* at single radius $R < R_0$
and $\Omega(R) > \Omega(R_0)$

What is $v_{\rm los}$ vs ℓ pattern?

A $v_{\rm los} = 0$ for all ℓ

B $v_{\rm los}$ changes across Galactic center
signal drops smoothly to nonzero minimum at anticenter

C $v_{\rm los}$ changes sign across Galactic center
but no signal at all for some ℓ

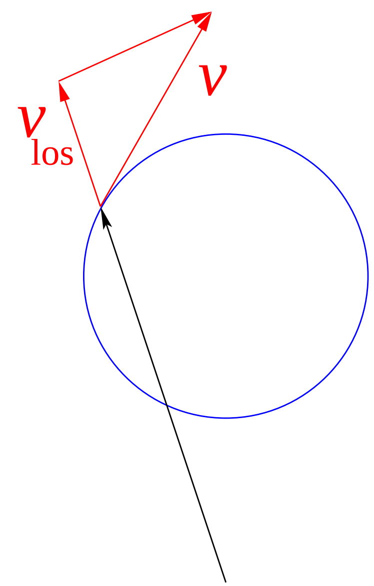
D $v_{\rm los}$ has same sign when signal nonzero
but no signal at all for some ℓ

E none of the above

One Inner Ring

features:

- $v_{\rm los}$ changes sign across Galactic center at thus is zero towards center at $\ell = 0$
- $v_{\rm los}$ maximum on sightline tangent to ring i.e., when $\ell_{\rm max}$ satisfies $R = R_0 \sin \ell_{\rm max}$
- for $|\ell| < \ell_{\rm max}$: signal from 2 points *Q: are $v_{\rm los}$ signs the same?*
- no gas found at $|\ell| > \ell_{\rm max}$ $\rightarrow$ no signal at these longitudes



Q: sketch of $v_{\rm los}$ vs ℓ?

Prediction: if *all* gas in *exterior ring* at $R > R_0$
and $\Omega(R) < \Omega(R_0)$ `www: outer ring sketch`
Q: what is rotational motion like? $v_{\rm los}$ *vs* ℓ *pattern?*

Prediction: think of gas disk as superposition of rings
if $\Omega(R)$ decreases with R,
Q: what is $v_{\rm los}$ *vs* ℓ *pattern?*

`www: velocity profile`