Astro 406
Lecture 29
Nov. 4, 2013

Announcements:

- **PS 9 due Friday**
- Planetarium makeup due Wednesday
- Midterm Exam–exam and scores posted tonite
  exams returned next time

Last time: lifestyles in an expanding universe

- cosmic scale factor $a(t)$
  Q: what is it? present value? past, future values?
- redshifts *Q: origin?*
- *"matter" definition for cosmologists?*
- *matter density versus a? evolution with time?*

Cosmic Density Evolution: Matter

matter for cosmo defined as *non-relativistic particles*
$\rightarrow \langle v \rangle \ll c$, $kT \ll mc^2$

if a non-relativistic species is not produced/destroyed
Q: when would this assumption break down?

- **number density** $n_{\rm matter} \propto a^{-3}$ (volume effect)

- **mass density** $\rho_{\rm matter} = m n_{\rm matter} \propto a^{-3}$
  $\rho_{\rm matter} = \rho_{\rm matter,0} \; a^{-3}$

- **energy density**: each particle has
  $E_{\rm tot} = E_{\rm rest} + E_{\rm kinetic} \approx mc^2 + mv^2/2 \approx mc^2$, so
  $\varepsilon_{\rm non-rel} = E_{\rm tot} n_{\rm non-rel} \approx mc^2 n_{\rm non-rel} = \rho_{\rm non-rel} c^2$

Cosmodynamics II

$a(t)$ gives expansion history of the Universe
which in turn tells how densities, temperatures change
$\rightarrow$ *given $a(t)$ can recover all of cosmic history!*

but...

How do we know $a(t)$?
What controls how scale factor $a(t)$ grow with time?
Q: what force(s) are at work?
Q: how are the force(s) properly described?

Cosmic Forces

- on microscale: particles scatter, collide
  via electromagnetic forces (also strong and weak forces)
  but no net charges or currents $\rightarrow$ no net forces

- pressure forces: manifestation of random velocities
  but pressure spatially uniform $\rightarrow$ *net* pressure force is *zero*!*
  Q: why uniform? why no net P force? (recall hydrostat eq)
  *See Director's Cut Extras; much more to come on cosmic pressure

- at large scales: only force is **gravity**

Q: what theory needed to describe this?

Cosmodynamics Computed

cosmic dynamics is evolution of a system which is

- gravitating
- homogeneous
- isotropic

Complete, correct treatment: General Relativity
solve Einstein's GR equations in homogeneous, isotropic Universe
⇒ to see this, take GR!

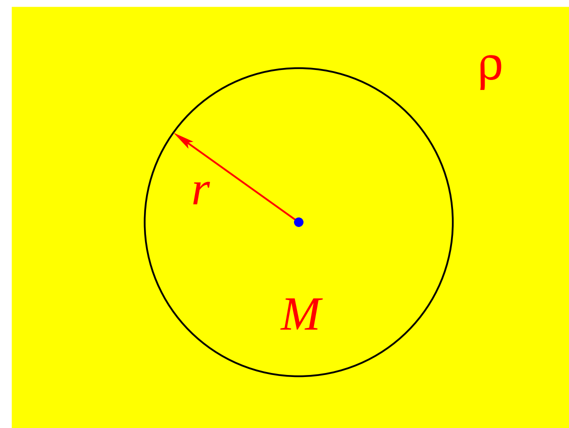
quick 'n dirty:
Non-relativistic (Newtonian) cosmology
pro: gives intuition, and right answer
con: involves some ad hoc assumptions only justified by GR

Inputs:

- arbitrary cosmic time t
- cosmic mass density is $\rho(t)$, spatially uniform *Q: why?*

Geometry:

pick an arbitrary point as origin $\vec{r} = 0$,
enclose in arbitrary sphere of radius $r(t)$:



enclosed mass $M(r) = 4\pi/3\ r^3\rho = const$
a point on the sphere feels acceleration
Q: what is $\ddot{r}$?

a point on the sphere feels acceleration

$$\ddot{r} = g = -\frac{GM(r)}{r^2} = -\frac{4\pi}{3} G\rho \; r \tag{1}$$

Q: why the − sign?

now introduce expansion technology:

- put $\boxed{r(t) = a(t) r_0}$
- for non-relativistic matter: $\rho = \rho_0 \; a^{-3}$, so

$$\ddot{a} = -\frac{4\pi}{3} G\rho_0 \; a^{-2} = -\frac{4\pi}{3} G\rho \; a \tag{2}$$

iClicker Poll: Pressureless Cosmic Matter Domination

in a pressureless, matter-only universe

$$\ddot{a} = -\frac{4\pi}{3} G\rho \; a \tag{3}$$

How must a matter-only universe evolve?

A it always expands

B it always contracts

C it always remains still or static

∞ D it can be static, but only for an instant a pressureless universe has **acceleration**

$$\ddot{a} = -\frac{4\pi}{3} G \rho_0 \, \frac{1}{a^2} \qquad (4)$$

formally identical to $\ddot{r} = -GM_{\rm earth}/r^2$

$\rightarrow$ motion of a *pop fly*: ball launched vertically

i.e., a ball moving upward in a radial orbit

scale factor a	$\Leftrightarrow$	ball height r
cosmic expansion rate $H = \dot{a}/a$	$\Leftrightarrow$	ball upward speed $v = \dot{r}$

pop fly: $\ddot{r} < 0$

state: *always* accelerated towards Earth

$\rightarrow$ "decelerated" compared to launch direction

fate: result of competition – *gravity vs inertia*

if gravity wins, $v = 0$ instantaneously at max height

then fall back down, ever faster

if inertia wins, v decreases, but never stops

Q: implications for pressureless, matter-only universe?

The Pop Fly Universe

pressureless, matter-only universe:
behavior very closely analogous to pop fly

pressureless, matter-only universe: $\ddot{a} = d\dot{a}/dt < 0$:

- cosmic expansion "speed" $\dot{a}$ *always* changing
  unless $\rho = 0$...
- expansion is *decelerated*

fate of this universe is result of competition:
gravity vs inertia

- *if gravity wins*, $H = 0$ instantaneously at max expand
  then *recollapse*, at ever greater rate
- *if inertia wins* H decreases, but *expand forever*

Why expansion?

the pop fly analogy is a very close one
and gives intuition that helps frame questions

such as: *initial conditions*
in the pop fly case, the *launch upwards*
sets the stage for the subsequent motion,
and is non-gravitational: David Ortiz's bat, or perhaps a rocket

so we ask: *what set the cosmic initial conditions?*
that is, *what drove the universe to expand?*
$\rightarrow$ this is a deep and open question!
probes conditions at earliest times (at or near "big bang")
we will discuss cosmic *inflation* as one answer to
What banged?

we found:

$$\ddot{a} = -\frac{4\pi}{3} G\rho_0 \; a^{-2} \tag{5}$$

multiply by $\dot{a}$ and integrate:

$$\dot{a} \; \frac{d}{dt}\dot{a} = -\frac{4\pi}{3} G\rho_0 \; \frac{\dot{a}}{a^2} \tag{6}$$

$$\dot{a} \; d\,\dot{a} = -\frac{4\pi}{3} G\rho_0 \; \frac{da}{a^2} \tag{7}$$

$$\frac{1}{2}\dot{a}^2 = \frac{4\pi}{3}\frac{G\rho_0}{a} + K \tag{8}$$

$$= \frac{4\pi}{3} G\rho a^2 + K \tag{9}$$

Q: look familiar?

this is an expression for $\dot{a}$

Q: physical significance?

The Friedmann Equation: I

recall that expansion rate $H = \dot{a}/a$

thus we can recast our cosmic "energy" equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2}$$

the **Friedmann equation**

Awesome!

- gives expression for expansion rate
  but need to know how ρ depends on a
  $\rightarrow$ *the expansion of the Universe depends on what's in it!*
- predicts cosmic past and future! *Q: how?*

Solving Friedmann: Matter Domination

important simple case: *matter-dominated universe*

- $\rho = \rho_{\text{matter}}$, dark matter included
- $K = 0$ (really: ρ term $\gg K$ term)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho \propto a^{-3} \quad (10)$$

$$\frac{\dot{a}}{a} \propto a^{-3/2} \quad (11)$$

$$a^{1/2} da \propto dt \quad (12)$$

integrate to find

$$a^{3/2} \propto t \quad (13)$$

$$a \propto t^{2/3} \quad (14)$$

this is huge! awesome! *Q: why?*

matter-only Friedmann $\rightarrow$ $a \propto t^{2/3}$

but we set $a(t_0) = 1$, so then $a_{\text{mat-dom}}(t) = (t/t_0)^{2/3}$

Huge! Gives history of (matter-dominated) universe!

Q: what does a plot of $a(t)$ look like?
Q: what it noteworthy about the $a(t)$ solution?
Q: what does it imply physically?

A Matter-Only Universe

$$\boxed{a(t) = (t/t_0)^{2/3}}$$

in this model: can relate z and t:

$$z(t) = \frac{1}{a(t)} - 1 = \left(\frac{t}{t_0}\right)^{-2/3} - 1 \tag{15}$$

- $t = t_0/2$ at $z = 2^{2/3} - 1 = 0.6$
- most distant QSO: $z \approx 7$
  corresponds to $t = t_0/8^{3/2} = 0.044\ t_0$
  "lookback time" is 96% of age of U

but remember–*these values only hold for matter-only universe*

Q: what about Hubble parameter $H(t)$ in this universe??

$$\boxed{a(t) = (t/t_0)^{2/3}}$$

in this model: can find *expansion rate*

$$H(t) = \frac{\dot{a}}{a} = \frac{2}{3}\,\frac{1}{t} \tag{16}$$

we see Hubble "constant" changes with time

Q: what is behavior of H over time?

Q: what does this imply for evolution of the U?

Q: what does this mean physically?

expansion rate $H(t) \propto 1/t$:
expansion slowing $\to$ U. decelerating
(i.e., $\ddot{a} < 0$)
gals: outward momentum opposed by inward gravity

Note:
Matter-only has

$$H_0 = \frac{2}{3}\frac{1}{t_0} \tag{17}$$

$$t_0 = \frac{2}{3}\frac{1}{H_0} = \frac{2}{3} t_{\rm Hubble} \tag{18}$$

predict t_0 from H_0: "expansion age"

Q: how can we use this connection to test cosmology?

Carding the Universe: Does the Age Check Out?

In any cosmo model: H_0 and t_0 related
but we can measure *both* values *independently*
$\rightarrow$ see if connection holds up

Since we know

$$H_0 = 72 \text{ km sec}^{-1} \text{ Mpc}^{-1} \tag{19}$$

we can solve for the "Hubble time"

$$t_{\rm Hub} = 13.8 \text{ billion years} = 13.8 \text{ Gyr} \tag{20}$$

for matter-only universe: expansion age $t_0 = \frac{2}{3} t_{\rm Hub} = 9.1$ Gyr

Oldest star (globular clusters) ages: $t_\star \sim 12 - 14$ Gyr, and $t_\star >$ 11.2 Gyr
thus we observe: $H_0 t_0 > 0.81$ and thus $\neq 2/3$
Q: what's going on?

Director’s Cut Extras

Pressure and Cosmic Evolution

in perfectly homogeneous universe, no pressure *differences*
$\rightarrow$ the usual pressure *forces* vanish

but recall an ideal gas: $P = nkT = 2n\langle KE \rangle/3$
$\rightarrow$ pressure $\propto$ kinetic energy density
and since $E = mc^2 \rightarrow$ all energy sources contribute to
total energy density $\varepsilon_{\rm tot}$ and thus
to equivalent mass density $\rho = \varepsilon_{\rm tot}/c^2$

non-relativistic matter: $\varepsilon_{\rm tot} \approx nmc^2$
$P = 0$; or really, take $P \ll nmc^2$
ideal gas: $P = nkT$, so $P/nmc^2 = kT/mc^2 \ll 1$

for *relativistic* particles, $\varepsilon_{\rm tot} \gg nmc^2$
$P/\varepsilon_{\rm tot}$ not small